HYDROGEN, FUEL CELL & ENERGY STORAGE

Hydrogen Utilization in Free Piston Engines: A Performance Investigation

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Abstract

This paper presents a dynamic analysis of a free-piston engine (FPE) using hydrogen gas as the working fluid for the first time. Initially, the governing dynamic equations of the FPE are derived. The performance of the B10-B engine is then evaluated based on the location of the closed-loop poles using five different working fluids: air, argon, nitrogen, helium, and hydrogen. The results indicate that hydrogen gas significantly enhances the engine's operating frequency, output power, and startup conditions compared to other gases. Additionally, this study examines the impact of varying key parameters of the FPE, such as power and displacer mass, the cross-sectional area of the pistons and the rod connected to the displacer piston, and the displacer piston stiffness on the engine's dynamics using the phase plane method. The findings reveal that the most optimal engine dynamics occur when hydrogen gas is used as the working fluid. Ultimately, the use of hydrogen as the working fluid leads to improved dynamic instability and increased output power, making it the optimal choice for FPEs, provided safety conditions are met to prevent combustion in the chamber.

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1 Introduction

Population growth and geopolitical risks, which have led to increased fossil fuel prices, have driven researchers to focus on the optimal use of renewable energies [1, 2]. Solar energy is one of the most promising alternatives to fossil fuels [3, 4]. The most common technology for generating electricity from solar energy is photovoltaic panels [5]. Recently, other technologies have been developed in this field. Among these, free-piston Stirling engines (FPSEs) are notable for their ability to convert solar energy into electricity through reciprocating motion [6, 7]. FPSEs were first designed and developed by Beale in 1964 [6]. Due to the elimination of mechanical links such as the flywheel and crankshaft, these engines have higher efficiency, operating frequency, and less mechanical friction compared than other types of Stirling engines [8, 9]. However, the dynamic structure of these engines presents a significant challenge for designers, particularly regarding startup conditions [10, 11].

Valuable research has been conducted to study the dynamics of FPSEs. Zare et al. [12] employed the practical stability method to investigate stable oscillations in an FPSE with air as the working fluid. Their study presented nine parametric conditions that, if satisfied, result in stable oscillations in the system's dynamics. Karabulut [13] studied the dynamics of an FPSE with air as the working fluid, evaluating the startup condition by numerically solving the governing equations and assessing changes in parameters such as the damping coefficient. Zare et al. [14] examined the presence or absence of a stable limit cycle in the nonlinear dynamics of an FPSE with air as the working fluid using the describing function method. This method not only predicts stable oscillations but also assesses the engine's performance. Formosa [15] investigated the performance of an FPSE using a semi-analytical technique with helium as the working fluid, calculating gas temperature to predict engine startup more accurately. Tavakolpour-Saleh and Zare [16] studied the nonlinear dynamics of an FPSE with air as the working fluid using the extended Lyapunov method, which examines stable oscillations using a general Lyapunov function.

Mou et al. [17] investigated the impact of the force exerted by the working fluid in the chamber of an FPSE on its performance, evaluating the effect of air at a pressure of 2 MPa on engine startup. Their findings showed that increasing the pressure of the working fluid improves engine startup. Part of the research conducted on the dynamics of Stirling engines has utilized neural networks (NNs). Deep learning methods utilize layered structures called NNs to mimic human behavior in making specific decisions based on data analysis [18]. The design of this layered structure is inspired by the human brain. Just as the human brain identifies patterns in data and categorizes various types of information, NNs can be trained in a similar way to recognize patterns and classify data. In other words, when the human brain encounters new information, it tries to compare it with its prior knowledge to gain a better understanding of the new data [19]. Similarly, the aim of NNs is to recognize patterns and classify new information based on their existing knowledge [20]. The process of learning these patterns occurs in the form of numerical vectors. In other words, all real-world data, such as images, sounds, and text, must be converted into numerical vectors and provided as input to the neural network so that the AI model can interpret them [21, 22].

Neural networks can model nonlinear problems, and because of this capability, they can be employed for a wide range of issues, such as free-piston Stirling engines. Significant research has been conducted in this area. In one of these studies, Zare et al. [23] reviewed and evaluated the research conducted on Stirling engines using neural networks. The results of this work showed that neural networks can be used in the evaluation of engine dynamics, startup, and the creation of a stable limit cycle, facilitating the design of such engines. Zare et al. [24] also evaluated the dynamics of a thermoacoustic Stirling engine (new version of FPSEs) using neural networks. The method presented in this work was able to accurately predict the engine's dynamic instability within an acceptable range. Ye et al. [25] have presented an ANN model for predicting the performance of a FPSE. In this study, the effects of six dynamic input parameters on the amplitude ratio, operating frequency, and phase angle were evaluated. Some output parameters were employed as training and testing data. The best outcomes were obtained from the 6-6-1, 6-6-1, and 6-10-6-1 network architectures for amplitude ratio, operating frequency, and phase angle, respectively. For these network architectures, the Levenberg-Marquardt backpropagation algorithm was used. The dynamic performance of the engine predicted by the neural network model was compared with the experimental data . After training, the correlation coefficient (R^2) amounts for both training and testing data were close to 1. The mean relative errors for amplitude ratio, operating frequency , and phase angle were 2.78%, 0.85% and 3.19% for the training process, respectively. These outcomes indicate that the ANN technique is an effective and reliable method for predicting the performance of a FPSE.

A review of previous research indicates that the study of FPSE dynamics has primarily been conducted with helium or air as the working fluid. However, the dynamics of these engines have not yet been investigated using other working fluids, such as hydrogen gas. Therefore, this paper examines the linear dynamics of FPSEs with air, argon, helium, nitrogen, and hydrogen as working fluids for the first time. The impact of changes in significant parameters such as power, displacer piston mass, cross-sectional area of pistons, and displacer piston stiffness on engine startup will be investigated.

As mentioned earlier, this paper will study and evaluate the dynamics of an FPSE using hydrogen as the working fluid for the first time. Section 2 pertains to the methodology, which includes the presentation and analysis of the linear dynamics governing the FPSE engine. Next, In section 3, the obtained results are analyzed and evaluated to compare the engine's performance based on different working fluids. Finally, the significant findings from this research are discussed (section 4).

2 Methodology

Figure 1 depicts a schematic representation of a FPSE. As shown in Figure 1, the FPSE consists of several components such as displacer and power pistons, regenerator, stiffness of the power and displacer pistons, and cylinder. The main role of the power piston is to transmit stable oscillations to the linear generator. On the other hand, the displacer piston is responsible for moving working fluid between the hot and cold chambers. The structure of the FPSE includes two mechanical springs to assist in the fluctuations of the pistons.

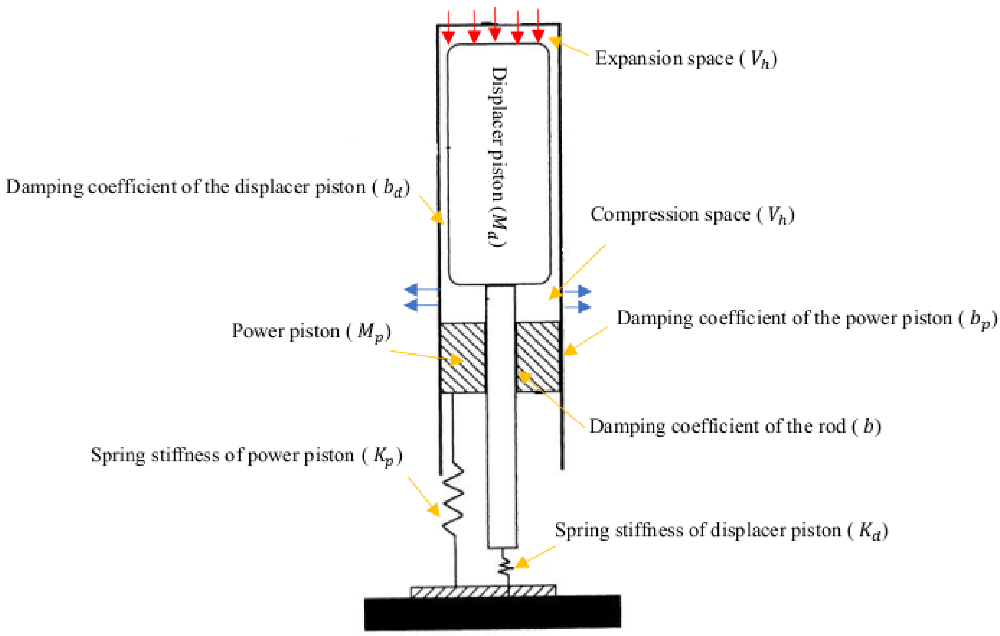


Fig. 1. Free piston Stirling engine.

According to Figure 1, the dynamic equations of the engine are as follows [7]:

$$M_p\ddot{y}_p + (b_p + b)\dot{y}_p - b\dot{y}_d + K_p y_p + \alpha y_p^3 + (P - P_0)(A - A_r) = 0\,, \quad (1)$$

$$M_d\ddot{y}_d + (b_d + b)\dot{y}_d - b\dot{y}_p + K_d y_d + \beta y_d^3 + (P - P_0)A_r = 0\,, \quad (2)$$

where

$$P - P_0 = (C_1 A - C_2 A + C_2 A_r)y_d + C_2(A - A_r)y_p\,, \quad (3)$$

$$C_{(1,2)} = MR\Big(\frac{V_{h_0}}{T_h} + \frac{V_{k_0}}{T_k}\Big)^{-2}\Big(\frac{1}{T_{(h,k)}}\Big)\,. \quad (4)$$

The linear dynamic equations of the FPSE in state-space form are as follows [26]:

$$\dot{x} = Ax + Bu\,, \quad (5)$$

$$\begin{bmatrix}\dot{y}_d\\ \ddot{y}_d\\ \dot{y}_p\\ \ddot{y}_p\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{-K_d}{M_d} & \frac{-(b_d+b)}{M_d} & 0 & \frac{b}{M_d}\\ 0 & 0 & 0 & 1\\ 0 & \frac{b}{M_p} & \frac{-K_d}{M_p} & \frac{-(b_p+b)}{M_p}\end{bmatrix}\begin{bmatrix}y_d\\ \dot{y}_d\\ y_p\\ \dot{y}_p\end{bmatrix} + \begin{bmatrix}0\\ \frac{A_r}{M_d}\\ 0\\ \frac{A-A_r}{M_d}u\end{bmatrix}, \quad (6)$$

where

$$u = -kx = -(P - P_0)\,, \tag{7}$$

$$u = -\begin{bmatrix} C_1A - C_2(A - A_r) & 0 & C_2(A - A_r) & 0 \end{bmatrix} \begin{bmatrix} y_d \\ \dot{y}_d \\ y_p \\ \dot{y}_p \end{bmatrix}. \tag{8}$$

By substituting Equation (8) into Equation (6), the closed-loop matrix governing the FPSE is obtained by Equation (9).

The eigenvalues of the closed-loop matrix (Equation (9)) are the closed-loop poles of the FPSE dynamic, which can be employed to study its dynamic behavior. It is important to note that the imaginary part of the dominant closed-loop poles of the system is equivalent to the operating frequency of the engine. The dynamic stability or instability of the engine is also determined using the real part of the closed-loop poles of the system. In other words, if the real part of the system's dominant closed-loop poles is positive, the system will be unstable; otherwise, it will be stable. Furthermore, the more positive the real part of the system's dominant closed-loop poles is, the better the engine's output power will be. The provided explanations can be schematically shown in Figure 2.

$$\begin{bmatrix} \dot{y}_d \\ \ddot{y}_d \\ \dot{y}_p \\ \ddot{y}_p \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_d}{M_d} - \frac{A_r}{M_d}[C_1A - C_2(A - A_r)] & -\frac{b_d+b}{M_d} & \frac{A_r}{M_d}C_2(A - A_r) & \frac{b}{M_d} \\ 0 & 0 & 0 & 1 \\ \frac{A-A_r}{M_d}[C_1A - C_2(A - A_r)] & \frac{b}{M_p} & -\frac{K_p}{M_p} + \frac{A-A_r}{M_d}C_2(A - A_r) & -\frac{b_p+b}{M_p} \end{bmatrix} \begin{bmatrix} y_d \\ \dot{y}_d \\ y_p \\ \dot{y}_p \end{bmatrix} \tag{9}$$

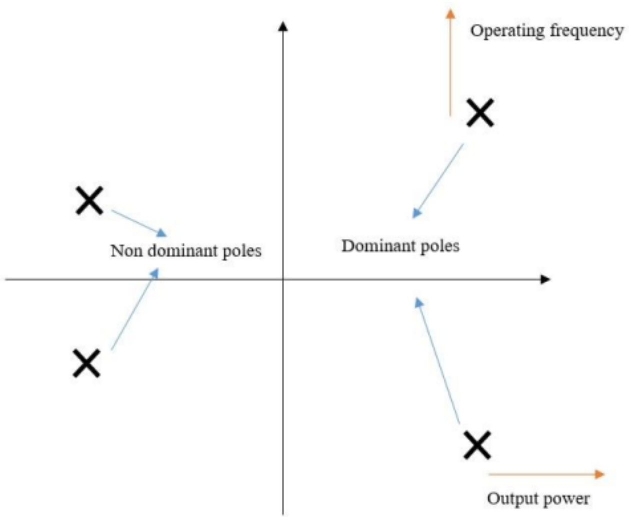


Fig. 2. The effect of dominant closed-loop poles on the operating frequency and output power of the engine.

It is worth mentioning that the engine's performance has been analyzed using the Root Locus method and state-space representation. Additionally, MATLAB software has been utilized for simulation purposes in this paper.

3 Results and Discussion

In this paper, the data of B10B (the FPSE manufactured by SUNPOWER) are used. The specifications of this engine are provided in Table 1.

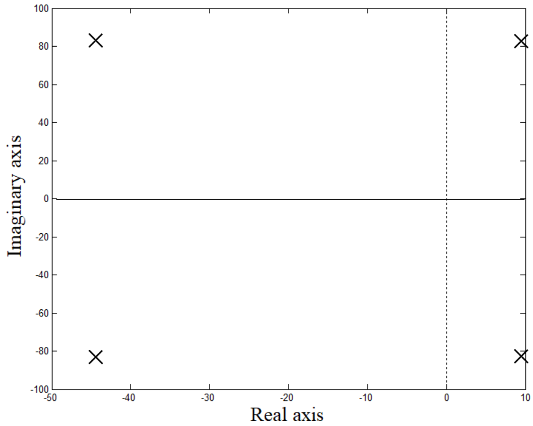
Table 1. Specifications of B10-B [18].

Parameter	Value
A	$0.00101\,\mathrm{m}^2$
M_d	0.086 kg
P_0	100 kPa
M_p	0.529 kg
M	4.58×10^{-5} kg
K_d	$600\,\mathrm{Nm}^{-1}$
K_p	$650\,\mathrm{Nm}^{-1}$

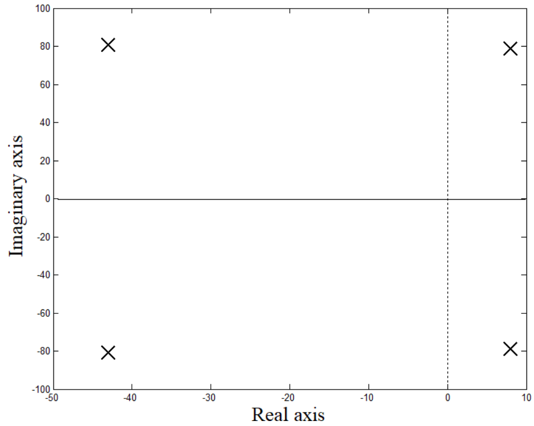
As stated, the main aim of this article is to study the performance of the FPSE based on different working fluids such as air, argon, nitrogen, helium, and hydrogen. According to Table 1 and Equation (9), the eigenvalues (closed-loop poles of the system) of the engine dynamics for each working fluid have been acquired and are depicted in Table 2. Additionally, Figure 3 demonstrates the location of the closed-loop poles of the system based on each of the working fluids (i.e., air, argon, nitrogen, helium, and hydrogen).

Table 2. Values of closed-loop poles based on working fluids.

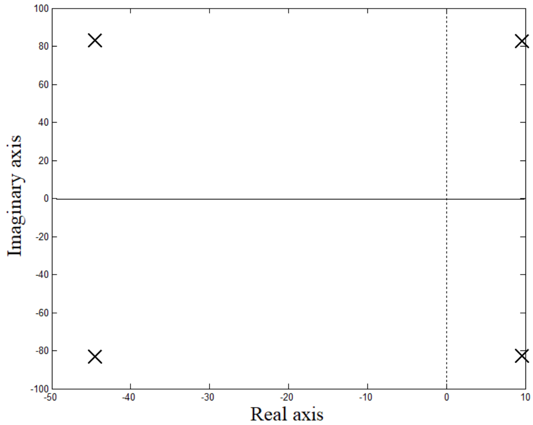
Working fluid	Dominant poles	Non-dominant poles
Air	$9.48 \pm 82.61j$	$-44.46 \pm 83.16j$
Argon	$8.01 \pm 78.71j$	$-42.99 \pm 80.93j$
Nitrogen	$9.55 \pm 82.81j$	$-44.53 \pm 83.29j$
Helium	$9.4 \pm 82.35j$	$-44.38 \pm 83.01j$
Hydrogen	$9.89 \pm 83.86j$	$-44.87 \pm 83.91j$



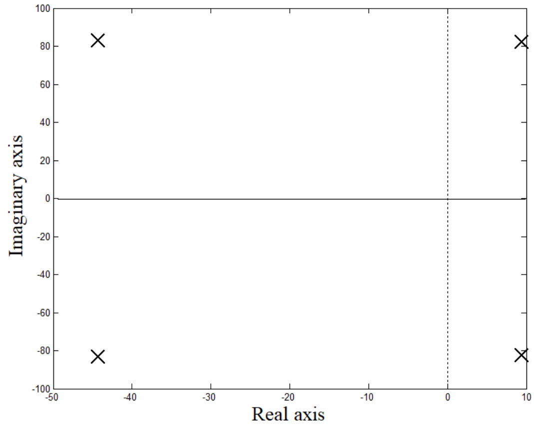
(a) Air



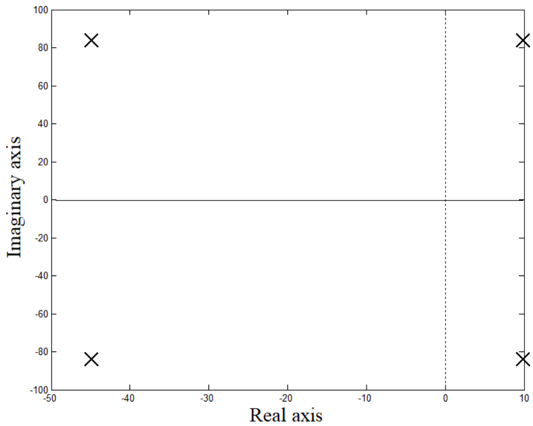
(b) Argon



(c) Nitrogen



(d) Helium



(e) Hydrogen

Fig. 3. Location of closed-loop system poles based on different working fluids in the phase plane.

Examination of the outcomes from Table 2 and Figure 3 reveals that using hydrogen gas as a working fluid (at a pressure of 1 bar) compared to other gases studied not only leads to an improvement in the operating frequency (the imaginary part of the dominant poles) of the FPSE but also increases its output power (the increase in the real part of the dominant pole). On the other hand, this work has shown that using hydrogen gas as the working fluid for the FPSE can result in better startup conditions (an increase in the real part of the dominant pole, which makes the system dynamic more unstable). With these explanations, using hydrogen gas as the working fluid can simultaneously bring three advantages to FPSEs.

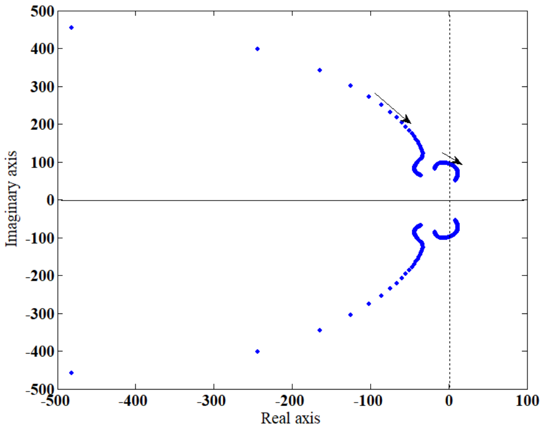
3.1 The effect of changing engine parameters on the value of the closed-loop system poles

Studying the effect of varying parameters of the FPSE on its performance can help in its optimal design and construction. Accordingly, this section will examine the impact of parameters such as the mass of power and displacer pistons (M_p, M_d), the displacer piston stiffness (K_d), and the cross-sectional area of the rod (A_r) and pistons (A) on the values of the dominant poles for each of the mentioned working fluids. In this regard, the pole locations will be plotted on the phase plane to study this subject.

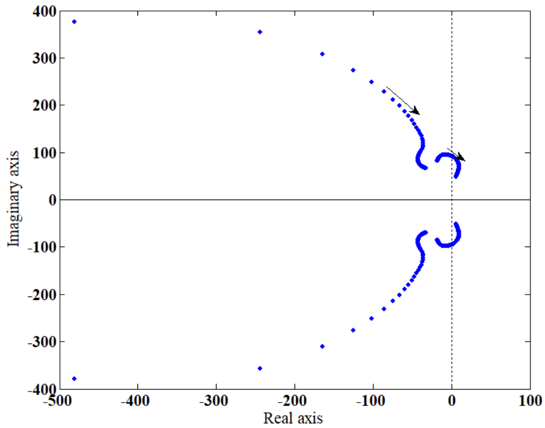
The power piston mass (M_p) is the first parameter whose impact on the engine dynamics will be studied for different working fluids (from 100 gr to 2 kg). Figure 4 shows the effect of varying the M_p on the dominant poles. The data extracted from figure shows that varying the M_p creates an optimal condition for the engine's dynamics. Table 3 presents the optimal values of the poles of the engine as a result of varying the M_p. As shown in Table 3 and Figure 4, the maximum real amount of the dominant poles of the system (which represents an increase in the output power of the oscillator) is higher when the working fluid is hydrogen compared to other gases (i.e., 11.7517).

Table 3. The optimal value of the dominant poles due to changes in the M_p.

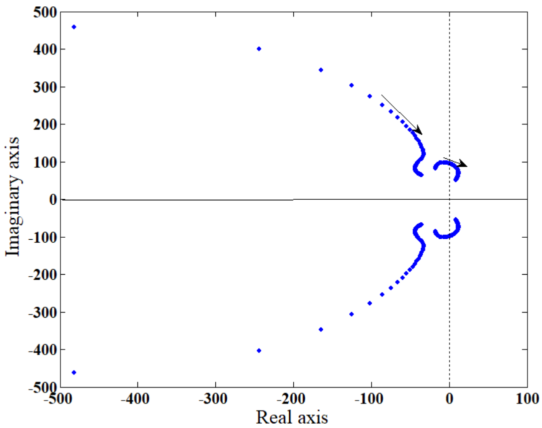
Working fluid	Dominant poles
Air	$11.0088 \pm 71.9217i$
Argon	$8.6959 \pm 71.3649i$
Nitrogen	$11.1311 \pm 71.8225i$
Helium	$10.8580 \pm 71.9810i$
Hydrogen	$11.7517 \pm 71.9149i$



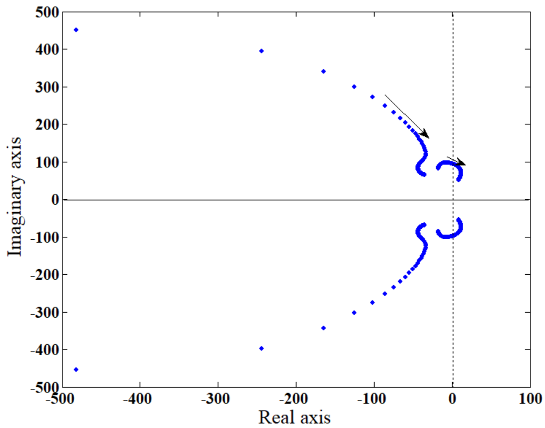
(a) Air



(b) Argon

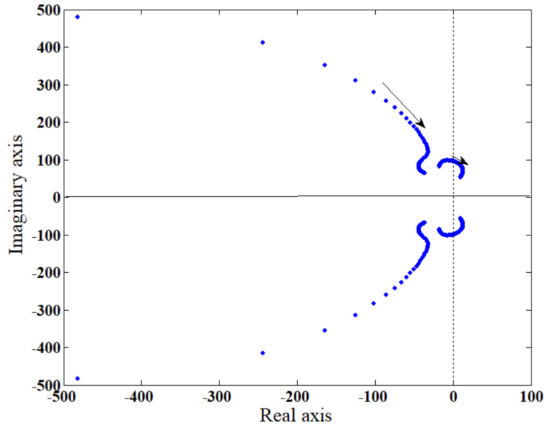


(c) Nitrogen



(d) Helium

Fig. 4. Effect of varying the M_p on the poles (to be continued).



(e) Hydrogen

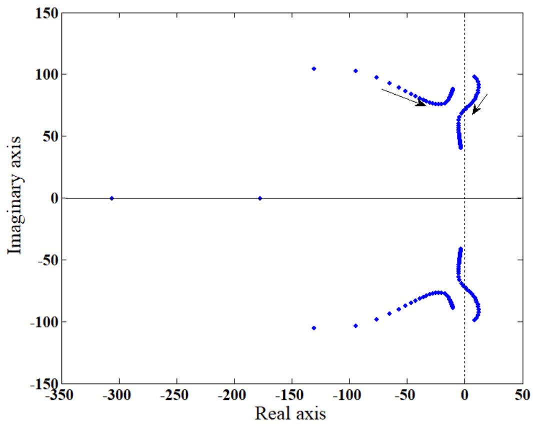
Fig. 4. Effect of varying the M_p on the poles.

Next, the effect of change in the displacer piston mass (M_d) on the location of the closed-loop poles of the system will be studied. Figure 5 shows the effect of varying the M_d on the poles of the system in the phase plane for each working fluid. As shown in Figure 5, a change in the M_d from 0.01 kg to 0.4 kg creates an optimal condition for the location of the dominant poles. Table 4 also presents the optimal amounts of the dominant poles for each of the working fluids employed in the engine (these values are obtained from Figure 5). The results of Table 4 and Figure 5 depict that when hydrogen gas is used as the working fluid, the real part of the optimal poles has its maximum value (i.e., 12.4598).

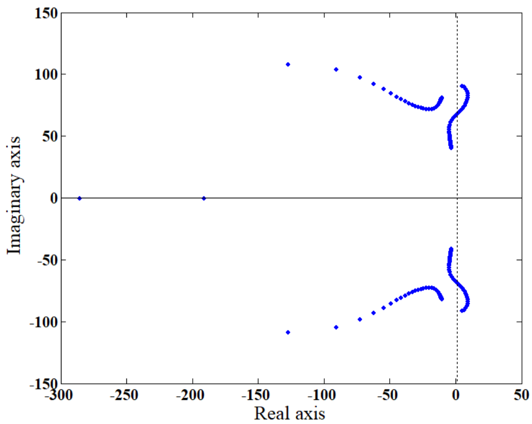
Figure 6 shows the effect of changes in the cross-sectional area (from $0.00005\,\mathrm{m}^2$ to $0.003\,\mathrm{m}^2$) of the pistons (A) on the values of the closed-loop poles of the system in the phase plane. Additionally, Table 5 shows the optimal values of the dominant poles of the engine based on Figure 6. The outcomes from Table 5 indicate that when hydrogen gas is employed as the working fluid in the engine chamber, the optimum engine performance (the highest real part value, i.e., 12.4598) is achieved.

Table 4. The optimal value of the dominant poles due to changes in the M_d.

Working fluid	Dominant poles
Air	$11.5090 \pm 89.7368i$
Argon	$8.7151 \pm 83.1524i$
Nitrogen	$11.6610 \pm 89.9827i$
Helium	$11.3211 \pm 89.4331i$
Hydrogen	$12.4598 \pm 93.6650i$

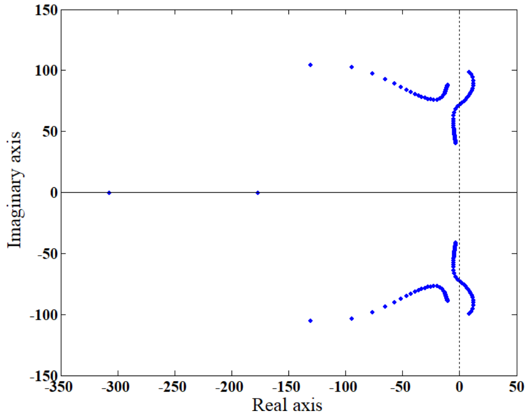


(a) Air

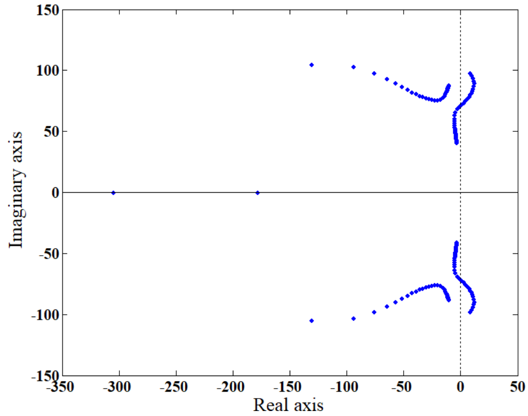


(b) Argon

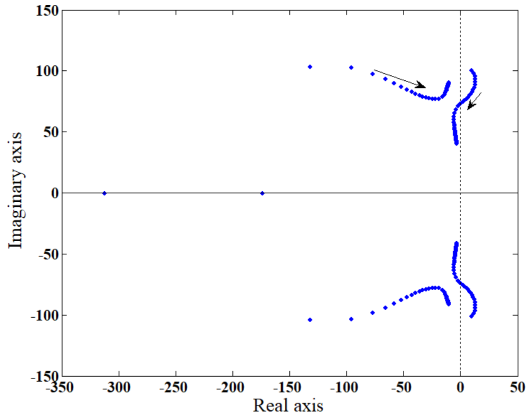
Fig. 5. Effect of varying the M_d on the poles (to be continued).



(c) Nitrogen



(d) Helium

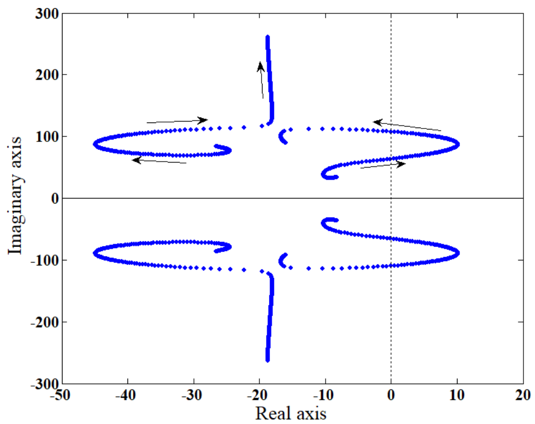


(e) Hydrogen

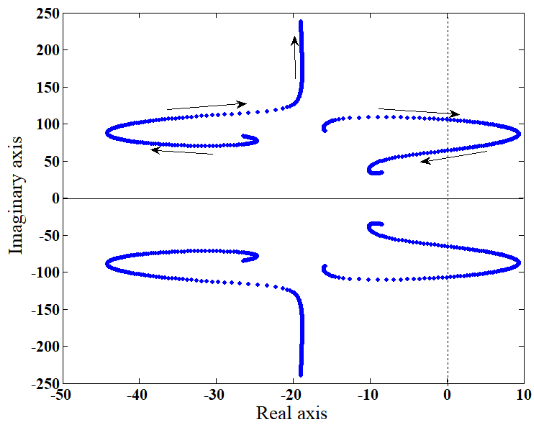
Fig. 5. Effect of varying the M_d on the poles.

Table 5. The optimal value of the dominant poles due to changes in the A.

Working fluid	Dominant poles
Air	$11.5090 \pm 89.7368i$
Argon	$8.7151 \pm 83.1524i$
Nitrogen	$11.6610 \pm 89.9827i$
Helium	$11.3211 \pm 89.4331i$
Hydrogen	$12.4598 \pm 93.6650i$

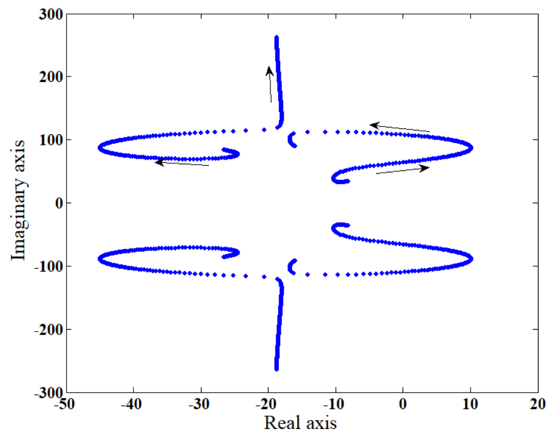


(a) Air

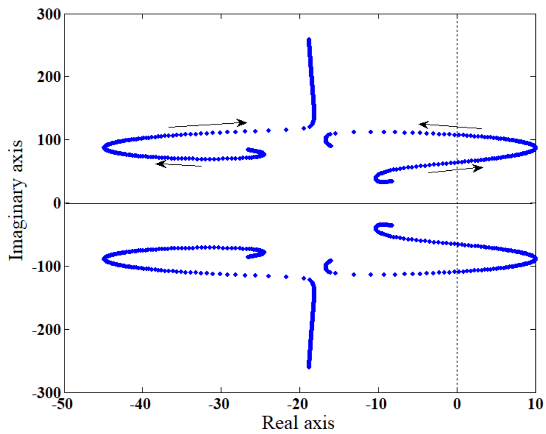


(b) Argon

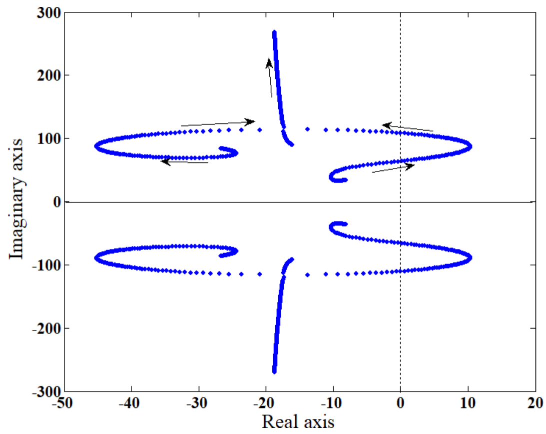
Fig. 6. Effect of varying the A on the poles (to be continued).



(c) Nitrogen



(d) Helium



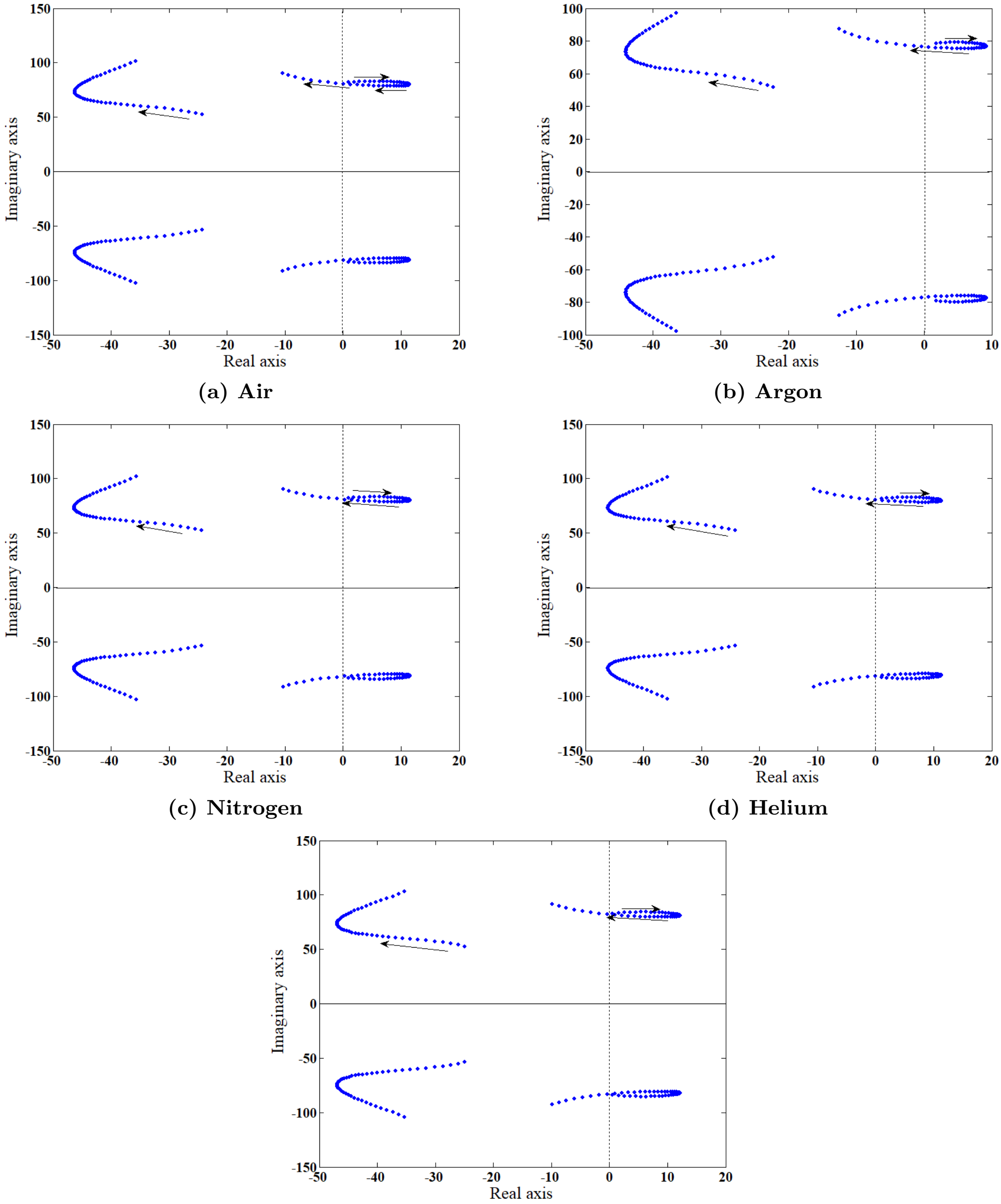
(e) Hydrogen

Fig. 6. Effect of varying the A on the poles.

Changes in the cross-sectional area of the displacer piston rod (A_r) are an important factor affecting the dynamic instability of FPSEs, and variations in this parameter create an optimal condition in the engine's dynamic behavior. The effects of this parameter (ranging from $0.0001\,\mathrm{m}^2$ to $0.0005\,\mathrm{m}^2$) for each of the working fluids on the values of the poles of the system have been investigated and evaluated, as shown in Figure 7. The optimal values of the real part of the dominant poles of the system for each working fluid are shown in Table 6. As indicated in Table 6, when hydrogen gas is employed as the working fluid, the real part of the dominant poles is at its highest (i.e., 12.0545). This means that using hydrogen gas results in increased output power and instability of the engine.

Table 6. The optimal value of the dominant poles due to changes in the A_r.

Working fluid	Dominant poles
Air	$11.3669 + 80.6125i$
Argon	$9.1279 \pm 77.2379i$
Nitrogen	$11.4793 \pm 80.8000i$
Helium	$11.2274 \pm 80.3814i$
Hydrogen	$12.0545 \pm 81.5223i$



(a) **Air** (b) **Argon** (c) **Nitrogen** (d) **Helium** (e) **Hydrogen**

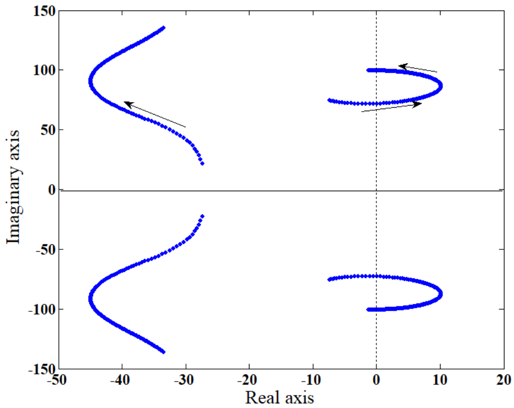
Fig. 7. Effect of varying the A_r on the poles.

The displacer piston stiffness (K_d) is the final parameter whose effect on the engine's dynamic is studied in this paper. Previous works have shown that the engine's dynamics are highly sensitive to the K_d, and even small changes can lead to stability or instability in the engine's dynamics. Accordingly, this work investigates the effect of varying the K_d from 100 N/m to 2000 N/m for each working fluid on the engine dynamic (Figure 8). The maximum real part of the poles of the system are also presented in Table 7. Table 7 shows that the most unstable condition for the engine's dynamics occurs when hydrogen gas is considered the working fluid (the value of the real part is obtained as 10.6829).

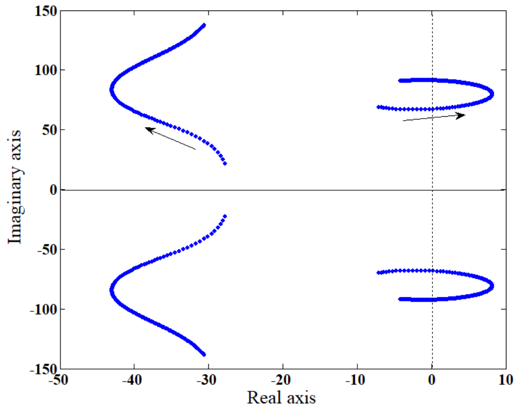
The analysis of the effect of varying engine parameters on the values of the poles of the system showed that using hydrogen gas as the working fluid improves dynamic instability (the main challenge for designers of this type of engine) and increases output power (by increasing the real part of the dominant poles). This shows that provided safety conditions are met (to prevent ignition in the chamber), hydrogen gas is the best option as a working fluid in FPSEs.

Table 7. The optimal value of the dominant poles due to changes in the K_d.

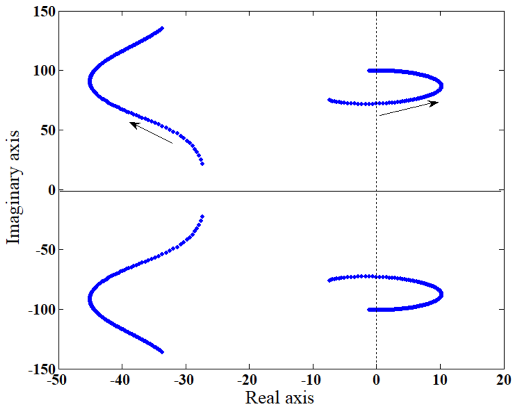
Working fluid	Dominant poles
Air	$10.0475 \pm 87.0225i$
Argon	$8.1073 \pm 80.4368i$
Nitrogen	$10.1519 \pm 87.2363i$
Helium	$9.9196 \pm 86.5304i$
Hydrogen	$10.6829 \pm 89.0043i$



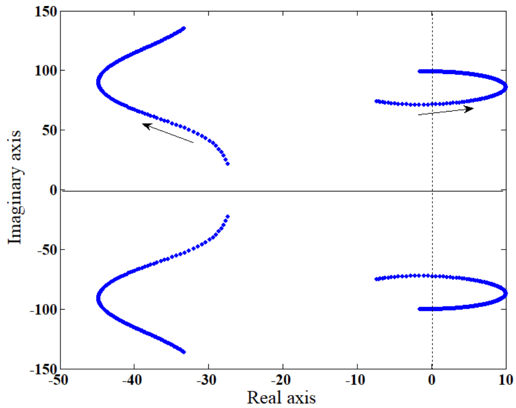
(a) Air



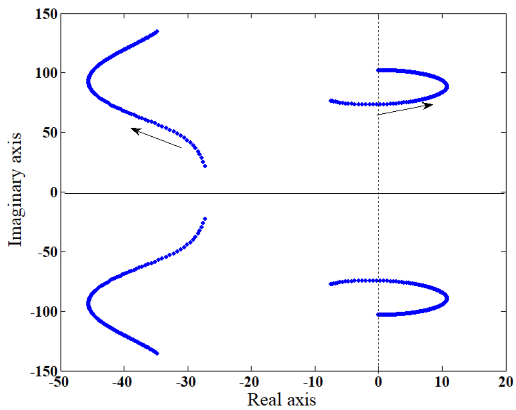
(b) Argon



(c) Nitrogen



(d) Helium



(e) Hydrogen

Fig. 8. Effect of varying the K_d on the poles.

4 Conclusion

This paper studied the performance of the FPSE using hydrogen gas (as the working fluid in the engine chamber) for the first time. Initially, the paper introduced the FPSE and the governing dynamic equations. Then, the performance of the B10-B engine (as a case study) was evaluated based on the location of the poles for five working fluids (i.e. air, argon, nitrogen, and hydrogen). The outcomes showed that if hydrogen gas is selected as the working fluid, the engine's operating frequency, output power, and startup conditions improve. This work studied the effect of varying important parameters of the FPSE, such as the power and displacer mass, the area of the pistons and the rod connected to the displacer piston, and the displacer piston stiffness, on the engine's dynamics using the phase plane. This investigation showed that changes in these parameters create an optimal condition in the engine's dynamic. In this regard, the results demonstrated that the most optimal condition (highest real part of the dominant poles) occurs when hydrogen gas is considered the working fluid. Overall, the analysis of the effect of varying engine parameters on the values of the system's closed-loop poles revealed that using hydrogen gas as the working fluid improves dynamic instability (the main challenge for designers of this type of engine) and increases output power (by increasing the real part of the dominant poles). In conclusion, the main contributions of this work are presented as follows:

- The dynamics of the FPSE were examined based on five working fluids.
- The operating frequency of the engine was analyzed and evaluated based on different working fluids using the poles of the system.
- The engine's output power was analyzed and evaluated based on different working fluids using the poles of the system.
- The superiority of using hydrogen as a working fluid over other fluids in enhancing the dynamic instability of the engine was examined and proven.

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