

Impact of Solar Radiation on Hybrid Nanofluid Dynamics in Rotating Microchannels

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Abstract

This study examines solar radiation effects on hybrid nanofluid behavior in a rotating microchannel. Governing nonlinear partial differential equations were transformed into dimensionless ordinary differential equations using similarity variables and solved analytically via the Hybrid Analytical and Numerical Method (HAN-method). Results indicate that increasing the rotation parameter reduces average fluid velocity in the x - and y -directions while enhancing z -direction velocity. Greater fluid suction at the upper wall elevates the surface friction coefficient and Nusselt number at the lower wall. Higher Reynolds numbers decrease temperature while increasing z -axis velocity, with corresponding increases in Nusselt number and surface friction coefficient at the bottom wall. Augmented solar radiation and temperature ratio parameters increase both Nusselt number (lower wall) and average fluid temperature. This research provides quantitative analytical solutions and introduces a novel validation methodology, representing a significant advancement over prior approaches.

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1 Introduction

Solar radiation is a green energy source because it is renewable, abundant, and has minimal environmental impact. It doesn't release greenhouse gases and can potentially meet all our future energy needs. It also involves radiant cooling and solar heating but can lead to undesired cooling or overheating due to its passive nature. Recent research prioritized developing switchable radiant systems for stable, efficient thermal management. Hu et al. [1] reviewed radiative sky cooling applications in solar energy and noted that Earth's dynamic energy balance involved both solar harvesting and radiative cooling. They also reported that mainstream solar technologies (PV, PT, PV/T, and CSP) have made significant progress, while advances in radiative cooling have largely stemmed from breakthroughs in materials science. Although solar collection and long-wave dissipation directions differed, combining them in one system proved possible. Radiative cooling increased PV efficiency by lowering temperatures, extended PT/PV/T operation to nighttime, and enhanced CSP efficiency by reducing condensing temperatures. The review discussed advancements and concluded with challenges and opportunities for combining these energies, aiming to summarize the status and guide future developments.

Yoo et al. [2] examined radiative thermal management using nano/microstructures. The findings revealed that passive radiative cooling and solar heating operated continuously, potentially causing undesired energy consumption during extreme temperatures. To overcome this, studies focused on technologies that could toggle radiative cooling or possess switchable dual modes. Their review explored fundamental concepts, switching mechanisms using novel materials/structures, and future directions. Fröhlich and Lean [3] explored solar electromagnetic radiation, Earth's primary energy source. Space-based measurements established variability's nature and origins. An 11-year cycle (0.1% amplitude) and larger 27-day variations (0.2%) were well-established in total solar irradiance (TSI). All spectral regions participated, with greater changes at shorter wavelengths. Variability is clearly linked to solar magnetism: active regions altered radiance, and their wavelength-dependent contrasts controlled the variability spectrum. TSI also responded to convection and eruptive events, exhibiting short-term fluctuations ($\sim 0.003\%$) and increasing during large flares ($\sim 0.015\%$). Multi-decadal changes potentially causing larger long-term variations remained unknown. Proxies suggested reduced TSI during the Maunder Minimum, but uncertainties pre-

cluded definitive historical estimates. Dai et al. [4] investigated high-temperature solar receivers, where efficiency was crucial as re-radiation loss increased dramatically with temperature. The fixed-bed Particle Solar Receiver (PSR) represented a promising pathway to high efficiency. Solar radiation penetrated deeper into its semi-transparent particles, enhancing the volumetric effect. Optimized PSRs achieved $\sim 92\%$ efficiency above 1000 K outlet temperature and maintained > 1000 K for $> 65\%$ of annual operating hours in CSP systems. Radiation energy was classified as incident solar radiation and radiative heat exchange to understand heat transfer and efficiency; mechanisms, solution methods, and progress for each were detailed. Challenges and future directions (design, experimentation, simulation) were proposed to advance PSRs for high-temperature power cycles.

Three-dimensional flow is critical due to its significant impact on fluid dynamics, heat transfer, and practical applications. Understanding these intricate patterns is essential for designing efficient aircraft, optimizing combustion processes, and predicting weather phenomena. Moreover, Gabriel et al. [5] tackled coastal hydrodynamics by developing a 3D numerical model using smoothed particle hydrodynamics (SPH). Their goal was to simulate complex near-shore currents – which are often shallow and turbulent – flowing over rough coastal terrain. Building on the open-source SPHysics platform, they introduced two key upgrades: first, a modified turbulence model (sub-particle-scale eddy viscosity) to better capture flow turbulence, and second, a drag force equation added to momentum calculations to account for roughness along the seabed and banks. To validate their model, they stacked their simulations against actual lab measurements across various flow conditions, comparing velocity, shear stress, and water surface behavior. Their results demonstrated that this enhanced SPH approach could accurately recreate realistic 3D near-shore currents, including surface effects, over rough topography.

Xuan and Shen [6] developed a convolutional neural network (CNN) to reconstruct 3D turbulent flows beneath free surfaces using only surface elevation and velocity measurements. They trained the model on datasets from direct numerical simulations of turbulent open-channel flows, enabling it to accurately reconstruct near-surface flow fields and capture large-scale subsurface structures. Their CNN significantly outperformed the traditional linear stochastic estimation (LSE) method in accuracy (measured by normalized mean squared and scale-specific errors). They further analyzed the CNN's saliency maps and LSE's kernels, revealing how both models utilize surface features and highlighting the importance of surface variables

for subsurface reconstruction. They also demonstrated the CNN's strong generalization: models trained on high-Froude-number flows successfully predicted flows at lower Froude numbers. Their results proved CNNs effective for detecting subsurface structures and – through interpretation of surface-subsurface relations – positioned them as promising tools for advancing physical understanding of free-surface turbulence.

Encinar and Jiménez [7] identified causally significant flow patterns in 3D isotropic turbulence using Jiménez's [8] perturbation method. They found significance depended on scale: kinetic energy dominated at large scales and enstrophy/dissipation at small scales. Significant regions contained strong gradients/energy and complex vortex clusters, while insignificant regions were weak with simple vortex sheets. Strain propagated perturbations more effectively than vorticity. Their results showed flow control should target strain-dominated clusters, avoiding sheets – validated by a simulation test. These insights aid turbulence control and prediction from limited data. Salimipour [9] investigated the interplay of three-dimensionality, thermal buoyancy, and vortex shedding on flow and heat transfer around a horizontal isothermal cylinder. Both buoyancy-induced secondary flow and free stream were upward. Simulations were conducted using Prandtl number 0.7, Reynolds numbers 100–300 (cylinder diameter-based), Richardson numbers 0–0.5, and aspect ratio 6. The Navier-Stokes, energy, and k - ω SST turbulence equations were solved via a finite-volume/pressure-based method with second-order accuracy (explicit for momentum/energy/turbulence, implicit for pressure).

Hu et al. [10] also investigated the effect of surface roughness on microscale flow, a common feature in microfluidic devices due to fabrication or particle adhesion. They developed a three-dimensional finite-volume numerical model to simulate pressure-driven liquid flow in microchannels with rectangular prism rough elements. They examined both symmetrical and asymmetric roughness arrangements, focusing on their influence on pressure drop. Their 3D simulations revealed significant impacts of roughness element height, size, spacing, and channel height on velocity distribution and pressure drop. They analyzed the compression-expansion flow around roughness elements and flow blockage within the channel. Finally, they presented an expression for the relative channel height reduction caused by roughness effects.

Microchannels play a pivotal role in various studies due to their unique characteristics, offering advantages for fluid control in microfluidics, heat transfer in microheat exchangers, and cell migration observation. Kadam et al. [11] investigated vapor bubble dynamics

critical to boiling inception in microchannels. They reviewed the full bubble lifecycle – nucleation at active sites, growth, departure, and motion – in submicron channels, emphasizing its impact on thermal/hydraulic behavior. They compiled existing models for the critical cavity radius needed for nucleation, noting significant deviations between them. Existing bubble growth models were also compiled; they concluded a more generalized model accounting for both inertia-controlled and thermal diffusion-controlled regions was needed. They analyzed forces (surface tension, inertia, shear, gravity, evaporation momentum) acting on growing bubbles and proposed that force analysis could identify the macro-to-microchannel threshold. Finally, they reported the impact of operating conditions, geometry, and fluid properties on microchannel bubble dynamics.

Zhang et al. [12] systematically reviewed microscale flow and heat transfer to advance energy efficiency in compact heat exchangers, noting that reduced channel scales alter flow and thermal behavior in ways that diverge from conventional systems due to factors like surface roughness, axial heat conduction, and special fluid properties. They addressed unresolved disagreements in existing research by focusing on two core mechanisms: (1) microscale single-phase flow/heat transfer influenced solely by scale effects, and (2) unique phenomena arising from coupling microscale flow with phase-change fluids (e.g., pseudophase change). Their analysis of these multifactor interactions aimed to clarify flow structures and heat transfer mechanisms, providing foundational insights for designing optimized microchannel heat exchangers.

Kandlikar et al. [13] critically reviewed heat transfer and fluid flow in microchannels, highlighting six key areas for future research. They confirmed that gas flow heat transfer is understood at a fundamental level (quantifying slip velocity/temperature jump effects and viscous heating from shear work) but noted a critical lack of experimental validation, urging adoption of techniques like molecular tagging thermometry (MTT). For liquid flow, they established macroscale-like behavior but highlighted a need for performance enhancement. In flow boiling, they identified lower heat transfer coefficients and critical heat flux (CHF) as major hurdles requiring breakthroughs, alongside developing active closed-loop controls to suppress instabilities. They characterized microchannel condensation as a unique process needing advanced flow regime maps and models beyond annular flow to address intermittent flows and diverse geometries/fluids. Finally, they emphasized electronics cooling as the primary driver but advocated parallel efforts to develop high-performance microscale heat exchangers through enhanced configurations, nanostructures, and practi-

cal implementation. Saha and Celata [14] comprehensively reviewed heat transfer and pressure drop phenomena in flow boiling within microchannels. They presented a state-of-the-art assessment of existing research in this field. Their book examined investigations on both heat transfer and pressure drop characteristics during microchannel flow boiling. It discussed predictions, models, and correlations for these parameters in detail. Finally, they concluded the work by outlining areas requiring further research.

Hybrid nanofluids represent a novel type of fluid that combines the benefits of nanoparticles with a base fluid, resulting in improved heat transfer performance [15]. However, addressing stability, production, and material selection issues is paramount for their practical adoption [16,17].

Realistic modeling of natural phenomena frequently results in highly nonlinear differential equations with complex nonlinearities, variable coefficients, and challenging boundary conditions. While numerical methods are commonly used to approximate solutions, they often lack the precise theoretical insights of analytical solutions. The Hybrid Analytical and Numerical Method (HAN method) bridges this gap by first using numerical techniques to generate approximate solutions and then converting them into closed-form analytical expressions. This approach combines computational efficiency with the exactness and interpretability of analytical solutions, offering a powerful framework for complex nonlinear differential equations (NDEs).

Recent literature highlights the HAN method's potential to overcome NDE challenges. Studies demonstrate its ability to enhance computational efficiency while extracting precise analytical solutions for deeper theoretical insights. Pioneering research [18] applied HAN to heat/mass transfer, momentum transport, and electromagnetic systems with intricate nonlinearities. Jalili et al. [19] examined thermo-diffusion, electric fields, and nonlinear radiation in non-Darcy Casson fluid flow, simplifying governing PDEs into ODEs via similarity transformations (STs). Jalili et al. [20] analyzed MHD flow of Reiner-Rivlin fluid using Von Kármán STs and HAN. Besides, Jalili et al. [21] investigated nanomaterial thin films over rotating plates, employing modified AGM and HAN. Jalili et al. [22] probed micropolar fluids between porous walls, revealing parameter influences. Ahmadi Azar et al. [23] also solved the Emden-Chandrasekhar equation for stellar structures, introducing a dimensionless constant ($\alpha = 0.1719381834$) and predicting solar properties. Ahmadi Azar et al. [24] analyzed unsteady Casson MHD flow in expandable channels. Jalili et al. [25] examined micropolar fluid between stretching disks. Moreover, Jalili et al. [26] investigated micropolar flow

in porous disk systems. Jalili et al. [27] derived exact solutions for von Kármán swirling flow with suction. Ahmadi Azar et al. [28] applied HAN to MHD hyperbolic tangent nanofluid flow over a wedge. Jalili et al. [29] analyzed oscillatory flow in wavy porous channels. Jalili et al. [30] developed an analytical approach for Taylor–Couette flow. Heidari et al. [31] solved the Klein-Gordon equation with quantum gravity modifications. Ahmadi Azar et al. [32] quantified temperature-dependent Casson fluid dynamics in porous channels. Ahmadi Azar [33] revisited Von Kármán flow using HAN. Ahmadi Azar [34] analyzed dusty hyperbolic tangent fluid flow with radiation effects. Finally, the hyperbolic tangent nano-fluid flow with radiation effects in porous wedge was investigated [35].

This study explores the steady alumina-copper/water hybrid nanofluid flow and heat transfer within a three-dimensional rotating microchannel. The microchannel features parallel walls, with the bottom wall being non-porous and stretching and the upper wall being porous and non-stretching. Both walls rotate simultaneously at a specific angular velocity, resulting in a three-dimensional flow with three velocity components. Additionally, solar radiative heat flux is considered in the partial constitutive differential equations. The governing PDEs are transformed into dimensionless nonlinear (ODEs) sets using similarity variables and non-dimensionalization. Then, these equations are solved analytically using the HAN method, and the correctness of the solutions is proven using a more innovative approach. This new approach has been used previously to prove the correctness of the results of a study. The results of that study (numerical or analytical) were compared with those of a previous similar study. The effects of the various parameters on velocity, temperature, skin friction coefficient, and Nusselt number are analyzed through appropriate graphs. According to the governing equations, some physical parameters depend on the fluid type, and others rely on the physics of the problem. Changing parameters, including injection/suction parameter, rotation parameter, Reynolds number, temperature ratio parameter, and radiation parameter were investigated in this study.

2 Mathematical Formulation

Let us consider the alumina-copper/water hybrid nanofluid flows between two rotating parallel plates, which these two plates construct a three-dimensional microchannel. As shown in Figure 1, two plates rotate simultaneously with the angular velocity vector $\vec{\Omega} = (0, \Omega, 0)$. In this microchannel, the lower wall is located in $y = 0$, and it is being stretched in the x -

direction with the stretching rate $U_w = bx$ in which b is a positive value ($b > 0$), but fluid suction/injection is not performed on it. This is while the upper wall is located in $y = h$ (height of the microchannel is h), and it is not under stretching, but fluid suction/injection is done from its perforated surface. The rate of the fluid injection from the upper surface is $-v_w$ and for the fluid suction is v_w . According to Figure 1, a forced convectional heat transfer occurs due to the temperature difference between two walls and the solar radiative heat flux. The temperature of the upper wall is T_0 while the bottom one is T_h ($T_h > T_0$).

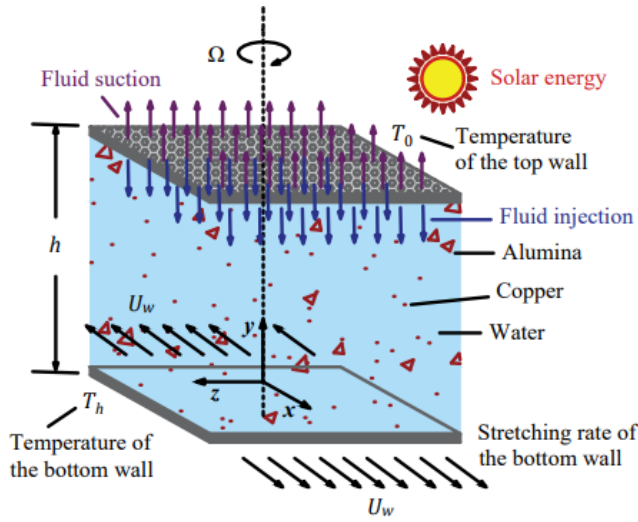


Fig. 1. The schematic of the problem.

For the mathematical modeling of Figure 1, the constitutive equations of continuity, momentum, and energy can be utilized in the Cartesian coordinate system as follows [36, 37]:

The conservation of mass (continuity):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (1)$$

The conservation of momentum along x-axis:

$$\rho_{\text{hnf}} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + 2\Omega w \right) = -\frac{\partial P^*}{\partial x} + \mu_{\text{hnf}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right). \quad (2)$$

The conservation of momentum along y-axis:

$$\rho_{\text{hnf}} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P^*}{\partial y} + \mu_{\text{hnf}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right). \quad (3)$$

The conservation of momentum along z-axis:

$$\rho_{\text{hnf}} \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} - 2\Omega u \right) = \mu_{\text{hnf}} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right). \quad (4)$$

The conservation of energy:

$$(\rho c_p)_{\text{hnf}} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k_{\text{hnf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - \frac{\partial q_r}{\partial y}. \quad (5)$$

In Equations (1) to (5), $u = u(x, y)$, $v = v(x, y)$, and $w = w(x, y)$ are velocity components in the Cartesian coordinates system (x, y, z) , ρ_{hnf} is the density of the hybrid nanofluid, $P^* = P(x, y) - \frac{\Omega^2 x^2}{2}$ is the modified pressure distribution of the hybrid nanofluid. It is beneficial to note that lack of pressure gradient assures the hybrid nanofluid flow along x-axis. $T = T(x, y, z)$ is the temperature distribution of the hybrid nanofluid, μ_{hnf} is the dynamic viscosity of the hybrid nanofluid, $(\rho c_p)_{\text{hnf}} = \rho_{\text{hnf}}(c_p)_{\text{hnf}}$ is the volumetric heat capacity of the hybrid nanofluid, $(c_p)_{\text{hnf}}$ is the specific heat capacity of the hybrid nanofluid, k_{hnf} is the thermal conductivity of the hybrid nanofluid, and q_r is the solar radiative heat flux. It is also equal as follows:

$$q_r = -\frac{4}{3} \frac{\sigma^*}{\kappa^*} \frac{\partial(T^4)}{\partial y} = -\frac{16}{3} \frac{\sigma^*}{\kappa^*} T^3 \frac{\partial T}{\partial y} \quad (6)$$

where σ^* is the Stefan-Boltzmann constant, and κ^* is the mean absorption coefficient.

The thermophysical properties of the hybrid nanofluid, such as ρ_{hnf} , μ_{hnf} , $(\rho c_p)_{\text{hnf}}$, and k_{hnf} are defined in Equations (7) to (10) [38–40]. According to these equations, the solid volume fraction of the aluminum oxide and copper are denoted by $\phi_{\text{Al}_2\text{O}_3}$ and ϕ_{Cu} , respectively, the dynamic viscosity of the base fluid (water) is shown with the μ_{bf} , the thermal conductivity of the aluminum oxide, copper, and the base fluid are shown with the $k_{\text{Al}_2\text{O}_3}$, k_{Cu} , and k_{bf} , respectively, the specific heat at constant pressure of the aluminum oxide, copper, and the base fluid are shown with the $(\rho c_p)_{\text{Al}_2\text{O}_3}$, $(\rho c_p)_{\text{Cu}}$, and $(\rho c_p)_{\text{bf}}$, respectively, and the density of the aluminum oxide, copper, and the base fluid are shown with the $\rho_{\text{Al}_2\text{O}_3}$, ρ_{Cu} , and ρ_{bf} , respectively. The thermophysical properties of base fluid and solid nanoparticles are shown in Table 1.

Table 1. The base fluid and solid nanoparticle thermophysical properties [40].

Physical Properties	Base Fluid (water)	Al ₂ O ₃	Cu
c_p (J kg ⁻¹ K ⁻¹)	4179	765	385
k (W m ⁻¹ K ⁻¹)	0.613	40	401
ρ (kg m ⁻³)	997.1	3970	8933
μ (Pa.s)	8.90E-4	-	-

$$\rho_{\text{hnf}} = \phi_{\text{Al}_2\text{O}_3} \rho_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}} \rho_{\text{Cu}} + (1 - \phi_{\text{Al}_2\text{O}_3} - \phi_{\text{Cu}}) \rho_{\text{bf}}, \quad (7)$$

$$\mu_{\text{hnf}} = \frac{\mu_{\text{bf}}}{(1 - \phi_{\text{Al}_2\text{O}_3} - \phi_{\text{Cu}})^{2.5}}, \quad (8)$$

$$(\rho c_p)_{\text{hnf}} = (1 - \phi_{\text{Al}_2\text{O}_3} - \phi_{\text{Cu}}) (\rho c_p)_{\text{bf}} + \phi_{\text{Al}_2\text{O}_3} (\rho c_p)_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}} (\rho c_p)_{\text{Cu}}, \quad (9)$$

$$k_{\text{hnf}} = \frac{\frac{\phi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}} k_{\text{Cu}}}{\phi_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}}} + 2k_{\text{bf}} + 2(\phi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}} k_{\text{Cu}}) - 2(\phi_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}}) k_{\text{bf}}}{\frac{\phi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}} k_{\text{Cu}}}{\phi_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}}} + 2k_{\text{bf}} - (\phi_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}} k_{\text{Cu}}) - (\phi_{\text{Al}_2\text{O}_3} + \phi_{\text{Cu}}) k_{\text{bf}}} k_{\text{bf}}. \quad (10)$$

Table 2. The thermophysical properties of the hybrid nanoparticle of Al_2O_3 –Cu/water

Solid's volume fraction	μ_{hnf}	k_{hnf}	$(\rho c_p)_{\text{hnf}}$	ρ_{hnf}
0.001	$8.944656E-4$	1.005973	4161585	1008.009
0.01	$9.361055E-4$	1.060826	4113918	1106.188
0.09	$1.461690E-3$	1.653892	3690213	1978.892
0.1	$1.554766E-3$	1.744647	3637250	2087.980
0.25	$5.034600E-3$	3.972429	2842803	3724.300

So, according to Table 1, the thermophysical properties of the Al_2O_3 –Cu/Water can be determined for different values of the solids volume fraction that Elhag et al. [41] presented, and these values are shown in Table 2.

Table 2 shows that the solid volume fraction of the aluminum oxide and copper are assumed to be the same. In the current study, the solid volume fraction with the value of 0.25 is considered for all calculations.

The boundary conditions associated with Equations (1) to (5) are as follows:

$$\begin{aligned} u(x, 0) &= U_w = bx, & v(x, 0) &= 0, & w(x, 0) &= 0, \\ T(x, 0, z) &= T_h, & u(x, h) &= 0, & v(x, h) &= v_w, \\ w(x, h) &= 0, & T(x, h, z) &= T_0, \end{aligned} \quad (11)$$

where $v_w > 0$ represents the hybrid nanofluid suction scenario from the top wall and inversely $v_w < 0$ represents the hybrid nanofluid injection scenario from the top wall.

The following similarity transformation can reduce the PDEs (1)–(5) with their corresponding boundary conditions (11) into a simpler form [36, 37]:

$$\begin{aligned} \eta &= \frac{y}{h}, & u &= bx \frac{df(\eta)}{d\eta}, & v &= -bh f(\eta), \\ w &= bx g(\eta), & \theta(\eta) &= \frac{T - T_0}{T_h - T_0}, \end{aligned} \quad (12)$$

where η is the independent dimensionless similarity variable, $f'(\eta)$ is the dimensionless hybrid nanofluid velocity along x -axis, $f(\eta)$ is the dimensionless hybrid nanofluid velocity along y -axis, $g(\eta)$ is the dimensionless hybrid nanofluid velocity along z -axis, and $\theta(\eta)$ is

the dimensionless hybrid nanofluid temperature. The presented similarity variables in Equation (12) are satisfied in the continuity Equation (1) and by applying Equation (12) on Equations (2) to (5), the following system of fourth-order nonlinear ODEs will be determined as below:

$$\begin{aligned} F(\eta) &:= \frac{d^4 f(\eta)}{d\eta^4} - \text{Re} \frac{L_1}{L_4} \left(\frac{df(\eta)}{d\eta} \frac{d^2 f(\eta)}{d\eta^2} - f(\eta) \frac{d^3 f(\eta)}{d\eta^3} \right) \\ &\quad - 2\lambda \frac{L_1}{L_4} \frac{dg(\eta)}{d\eta} = 0, \end{aligned} \quad (13)$$

$$\begin{aligned} G(\eta) &:= \frac{d^2 g(\eta)}{d\eta^2} - \text{Re} \frac{L_1}{L_4} \left(g(\eta) \frac{df(\eta)}{d\eta} - f(\eta) \frac{dg(\eta)}{d\eta} \right) \\ &\quad + 2\lambda \frac{L_1}{L_4} \frac{df(\eta)}{d\eta} = 0, \end{aligned} \quad (14)$$

$$\begin{aligned} \Theta(\eta) &:= \frac{d^2 \theta(\eta)}{d\eta^2} + \frac{4N\text{Pr}}{3L_3} \frac{d}{d\eta} \left[(1 + \theta(\eta) (\theta_w - 1))^3 \frac{d\theta(\eta)}{d\eta} \right] \\ &\quad + \frac{L_2}{L_3} \text{Re} f(\eta) \frac{d\theta(\eta)}{d\eta} = 0. \end{aligned} \quad (15)$$

In ODEs (13)–(15), $\frac{d}{d\eta}$ is the total derivative operator with respect to the variable η , $\text{Re} = \frac{bh^2}{\nu_{\text{hnf}}}$ is the Reynolds number, $\text{Pr} = \frac{\mu_{\text{hnf}} (\rho c_p)_{\text{hnf}}}{\rho_{\text{hnf}} k_{\text{hnf}}}$ is the Prandtl number, $\lambda = \frac{\Omega h^2}{\nu_{\text{hnf}}}$ is the rotation parameter, $N = \frac{4\sigma^* T_0^3}{\kappa^* k_{\text{hnf}}}$ is the solar radiation parameter, $L_1 = \frac{\rho_{\text{hnf}}}{\rho_{\text{bf}}}$, $L_2 = \frac{(\rho c_p)_{\text{hnf}}}{(\rho c_p)_{\text{bf}}}$, $L_3 = \frac{k_{\text{hnf}}}{k_{\text{bf}}}$, $L_4 = \frac{\mu_{\text{hnf}}}{\mu_{\text{bf}}}$, and $\theta_w = \frac{T_h}{T_0}$

is the temperature ratio parameter, and in this study, θ_w is always considered greater than 1 ($\theta_w > 1$).

To derive the boundary conditions of ODEs (13)-(15), the similarity variables (12) should be placed in the boundary conditions (11) to obtain the boundary conditions related to ODEs (13)-(15) as follows:

$$\begin{aligned} f(0) &= 0, & f'(0) &= 1, \\ g(0) &= 0, & \theta(0) &= 1, \\ f(1) &= S, & f'(1) &= 0, \\ g(1) &= 0, & \theta(1) &= 0, \end{aligned} \quad (16)$$

where the suction/injection parameter is $S = \frac{v_w}{bh}$. The hybrid nanofluid suction from the top wall is related to when the $S > 0$, injection from the top wall is related to when the $S < 0$, and when the top wall is impermeable (solid) is related to when the $S = 0$. There are two physical quantities called the skin friction coefficient and the Nusselt number, which are very useful in engineering and are defined as follows for the bottom

wall of the microchannel [37]:

$$C_f = \frac{2\tau_w}{\rho_{\text{hnf}} U_w^2}, \quad (17)$$

$$\text{Nu} = \frac{hq_w}{k_{\text{bf}}(T_h - T_0)}. \quad (18)$$

where τ_w and q_w , are the lower wall's shear stress and heat transfer rate, respectively, and they are defined as follows [37]:

$$\tau_w = \mu_{\text{hnf}} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad (19)$$

$$q_w = -k_{\text{hnf}} \left(\frac{\partial T}{\partial y} \right)_{y=0} + (q_r)_{y=0}. \quad (20)$$

For making the Equations (17) and (18) are simpler, and the similarity variables (12) are applied in Equations (19) and (20). The results of this substitution are applied to Equations (17) and (18). By doing this, the skin friction coefficient and Nusselt number can be demonstrated in terms of similarity variables:

$$C_f = \frac{(1 - \phi_{\text{Al}_2\text{O}_3} - \phi_{\text{Cu}})^{-2.5}}{(1 - \phi_{\text{Al}_2\text{O}_3} - \phi_{\text{Cu}}) + \phi_{\text{Al}_2\text{O}_3} \left(\frac{\rho_{\text{Al}_2\text{O}_3}}{\rho_{\text{bf}}} \right) + \phi_{\text{Cu}} \left(\frac{\rho_{\text{Cu}}}{\rho_{\text{bf}}} \right)} \frac{d^2 f(\eta)}{d\eta^2} \bigg|_{\eta=0}, \quad (21)$$

$$\text{Nu} = -\frac{k_{\text{hnf}}}{k_{\text{bf}}} \left[1 + \frac{4N}{3L_3} (1 + (\theta_w - 1)\theta(0))^3 \right] \frac{d\theta(\eta)}{d\eta} \bigg|_{\eta=0}. \quad (22)$$

It is obvious that Equations (21) and (22) can analyzed only by solving ODEs (13)-(15).

3 Application of the HAN Method

The HAN method has been employed in numerous studies to address nonlinear ordinary differential equations. It is recognized for providing a more accurate analytical solution with fewer terms than other semi-analytical methods. Three polynomials with constant coefficients were assumed as the analytical solutions to derive the semi-analytical solution for the system of ordinary differential equations using the HAN method. Thus, the following polynomials are the semi-analytical solutions of the ODEs (13)-(15):

$$f(\eta) = \sum_{i=0}^7 a_i \eta^i, \quad g(\eta) = \sum_{i=0}^5 b_i \eta^i, \quad \theta(\eta) = \sum_{i=0}^5 c_i \eta^i. \quad (23)$$

In the analytical solution of the Equations (13)-(15), there are 20 unknown coefficients. These coefficients

can be determined by solving a system of 20 algebraic equations. Algebraic equations can be constructed from the boundary conditions (16) of the problem as follows:

$$f(0) = \sum_{i=0}^7 a_i(0)^i = a_0 = 0, \quad (24)$$

$$f'(0) = \sum_{i=1}^7 i a_i(0)^{i-1} = a_1 = 1, \quad (25)$$

$$g(0) = \sum_{i=0}^5 b_i(0)^i = b_0 = 0, \quad (26)$$

$$\theta(0) = \sum_{i=0}^5 c_i(0)^i = c_0 = 1, \quad (27)$$

$$f(1) = \sum_{i=0}^7 a_i(1)^i = S, \quad (28)$$

$$f'(1) = \sum_{i=1}^7 i a_i(1)^{i-1} = 0, \quad (29)$$

$$g(1) = \sum_{i=0}^5 b_i(1)^i = b_0 = 0, \quad (30)$$

$$\theta(1) = \sum_{i=0}^5 c_i(1)^i = 0. \quad (31)$$

However, these algebraic equations (24)–(31) are insufficient because the order of ODEs is generally less than the degree of the assumed polynomials. Therefore, the remaining algebraic equations (12 algebraic

equations) can be made through the approximated boundary conditions. The approximated boundary conditions are made from the numerical solutions. Tables 3 and 4 show the numerical solution of ODEs (13)–(15) when the physical parameters are: $\phi_{\text{Al}_2\text{O}_3} = 0.25$, $\phi_{\text{Cu}} = 0.25$, $\rho_{\text{Al}_2\text{O}_3} = 3970$, $\rho_{\text{Cu}} = 8933$, $\rho_{\text{bf}} = 997.1$, $k_{\text{Al}_2\text{O}_3} = 40$, $k_{\text{Cu}} = 401$, $k_{\text{bf}} = 0.613$, $(c_p)_{\text{Al}_2\text{O}_3} = 765$, $(c_p)_{\text{Cu}} = 385$, $(c_p)_{\text{bf}} = 4179$, $\lambda = 2$, $\text{Pr} = 0.967411$, $\text{Re} = 2$, $N = 0.5$, $\theta_w = 1.5$, and $S = 1$.

Table 3. The numerical solution of the $f(\eta)$, $df(\eta)/d\eta$, $d^2f(\eta)/d\eta^2$, and $d^3f(\eta)/d\eta^3$ using the Maple.

η	$f(\eta)$	$df(\eta)/d\eta$	$d^2f(\eta)/d\eta^2$	$d^3f(\eta)/d\eta^3$
0.00	0	1	2.58353453311703	−10.8298716022412
0.20	0.238349795258861	1.32222053650246	0.741871129705633	−7.75195368578725
0.40	0.508035749401300	1.32988924081121	−0.605290129198973	−5.95186722804095
0.60	0.754275169401941	1.09553674779758	−1.71709950746895	−5.30391232282027
0.80	0.932055257058124	0.647743768792499	−2.75249010955764	−5.03898557885191
1.00	1	0	−3.70076904940068	−4.30254152826803

Table 4. The numerical solution of the $g(\eta)$, $dg(\eta)/d\eta$, $\theta(\eta)$, $d\theta(\eta)/d\eta$, using the Maple.

η	$g(\eta)$	$dg(\eta)/d\eta$	$\theta(\eta)$	$d\theta(\eta)/d\eta$
0.00	0	2.29756987650351	1	−0.8902466608316
0.20	0.35104348534303	1.17634248654101	0.81495454573411	−0.9586492765879
0.40	0.47111223484493	0.05698314662832	0.61756323216920	−1.0117407378258
0.60	0.39553624042495	−0.7426817360419	0.41199688810026	−1.0389214835218
0.80	0.20550536616341	−1.0739569678483	0.20410900521112	−1.0347003358624
1.00	0	−0.8958700718045	0	−1.0025738845577

In Tables 3 and 4, the bold numbers are related to the boundary conditions (16), and the non-bold numbers are related to the numerical solution of the Maple (the non-bold numbers are counted as the approximated boundary conditions). According to Tables 3 and 4, the following algebraic equations can be constructed:

$$f(0.2) = \sum_{i=0}^7 a_i(0.2)^i = 0.238349795258861, \quad (32)$$

$$f(0.4) = \sum_{i=0}^7 a_i(0.4)^i = 0.508035749401300, \quad (33)$$

$$f(0.6) = \sum_{i=0}^7 a_i(0.6)^i = 0.754275169401941, \quad (34)$$

$$f(0.8) = \sum_{i=0}^7 a_i(0.8)^i = 0.932055257058124, \quad (35)$$

$$g(0.2) = \sum_{i=0}^5 b_i(0.2)^i = 0.35104348534303, \quad (36)$$

$$g(0.4) = \sum_{i=0}^5 b_i(0.4)^i = 0.47111223484493, \quad (37)$$

$$g(0.6) = \sum_{i=0}^5 b_i(0.6)^i = 0.39553624042495, \quad (38)$$

$$g(0.8) = \sum_{i=0}^5 b_i(0.8)^i = 0.20550536616341, \quad (39)$$

$$\theta(0.2) = \sum_{i=0}^5 b_i(0.2)^i = 0.81495454573411, \quad (40)$$

$$\theta(0.4) = \sum_{i=0}^5 b_i(0.4)^i = 0.61756323216920, \quad (41)$$

$$\theta(0.6) = \sum_{i=0}^5 b_i(0.6)^i = 0.41199688810026, \quad (42)$$

$$\theta(0.8) = \sum_{i=0}^5 b_i(0.8)^i = 0.20410900521112, \quad (43)$$

By solving 20 unknown coefficients of the Equation (23) and 20 algebraic Equations (24)-(43), the semi-analytical solution of the ODEs (13)-(15) and its corresponding boundary conditions (16) is derived as below:

$$f(\eta) = 0.04305159376 \eta^7 - 0.03127422941 \eta^6 - 0.3069826168 \eta^5 + 0.8291581118 \eta^4 - 1.827529424 \eta^3 + 1.293576565 \eta^2 + \eta, \quad (44)$$

$$g(\eta) = -0.7309726120 \eta^5 + 2.656212728 \eta^4 - 1.720441164 \eta^3 - 2.510942350 \eta^2 + 2.306143398 \eta, \quad (45)$$

$$\theta(\eta) = -0.03738901236 \eta^5 + 0.1185973708 \eta^4 - 0.01803556601 \eta^3 - 0.1722224840 \eta^2 - 0.8909503084 \eta + 1, \quad (46)$$

The validity of the semi-analytical solutions (44)-(46) is proved by substituting these analytical solutions into ODEs (13)-(15). After substitution, three functions $F(\eta)$, $G(\eta)$, and $\Theta(\eta)$, are constructed, and the average value of these functions in the considered domain of the independent variable is expected to be approximately equal to zero. The results of this substitution are demonstrated in Figures 2-4.

The average values of $F(\eta)$ are 0.481291532817538, $G(\eta)$ is -0.0439033083879079 , and $\Theta(\eta)$ is 0.00428887147131670, which these average values are not approximately near zero. According to Figures 2-4, the most identified calculation errors are at the beginning and end points. Therefore, the assumed solution of Equation (23) is modified with increasing number of terms and unknown coefficients as below:

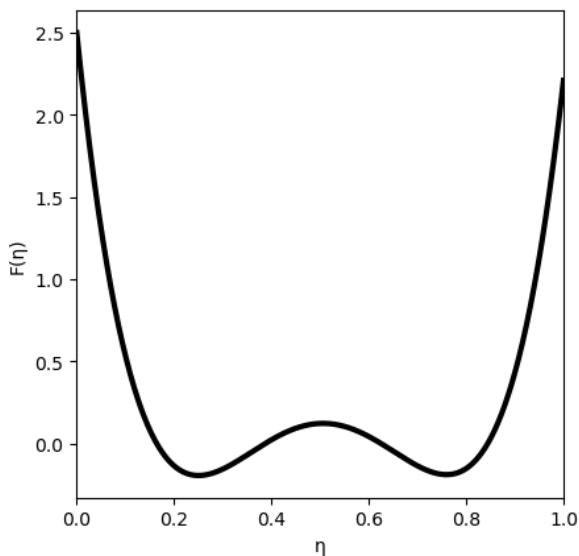


Fig. 2. The validity of $f(\eta)$ and $g(\eta)$ Solutions.

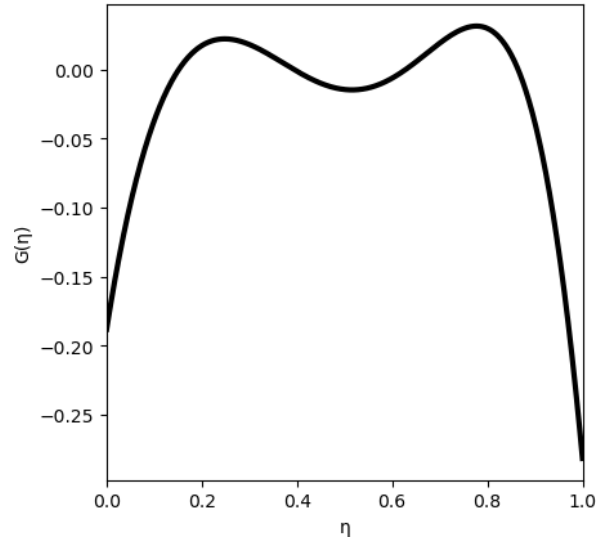


Fig. 3. The validity of $f(\eta)$ and $g(\eta)$ solutions.

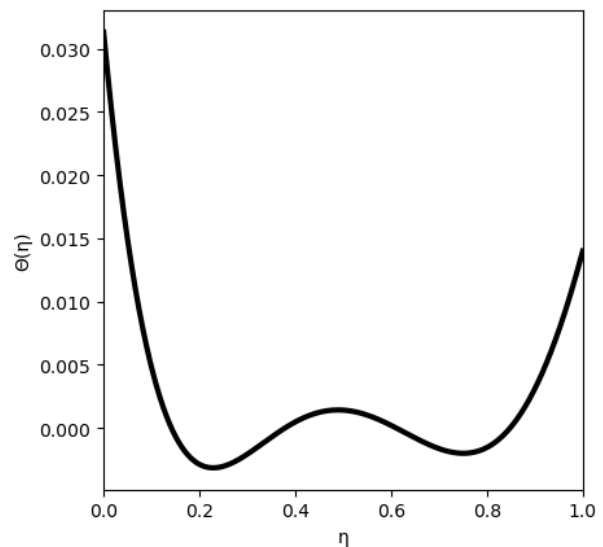


Fig. 4. The validity of $f(\eta)$ and $\theta(\eta)$ solutions.

$$\begin{aligned} f(\eta) &= \sum_{i=0}^{11} a_i \eta^i, \\ g(\eta) &= \sum_{i=0}^7 b_i \eta^i, \\ \theta(\eta) &= \sum_{i=0}^7 c_i \eta^i, \end{aligned} \quad (47)$$

where Equation (47) is the semi-analytical solution of the ODEs (13)-(15), it has 28 unknown coefficients and needs 28 algebraic equations to determine these coefficients. The boundary conditions of the problem (16)

can only make 8 algebraic equations as below:

$$f(0) = \sum_{i=0}^{11} a_i(0)^i = a_0 = 0, \quad (48)$$

$$f'(0) = \sum_{i=1}^{11} i a_i(0)^{i-1} = a_1 = 1, \quad (49)$$

$$g(0) = \sum_{i=0}^7 b_i(0)^i = b_0 = 0, \quad (50)$$

$$\theta(0) = \sum_{i=0}^7 c_i(0)^i = c_0 = 1, \quad (51)$$

$$f(1) = \sum_{i=0}^{11} a_i(1)^i = S, \quad (52)$$

$$f'(1) = \sum_{i=1}^{11} i a_i(1)^{i-1} = 0, \quad (53)$$

$$g(1) = \sum_{i=0}^7 b_i(1)^i = b_0 = 0, \quad (54)$$

$$\theta(1) = \sum_{i=0}^7 c_i(1)^i = 0. \quad (55)$$

It is obvious that the number of algebraic equations is not enough, and the remaining algebraic equations should be constructed through [Tables 3 and 4](#) as below:

$$\begin{aligned} f''(0) &= \sum_{i=2}^{11} i(i-1)a_i(0)^{i-2} \\ &= 2.58353453311703, \end{aligned} \quad (56)$$

$$\begin{aligned} f'''(0) &= \sum_{i=3}^{11} i(i-1)(i-2)a_i(0)^{i-3} \\ &= -10.8298716022412, \end{aligned} \quad (57)$$

$$g'(0) = \sum_{i=1}^7 i b_i(0)^{i-1} = 2.29756987650351, \quad (58)$$

$$\theta'(0) = \sum_{i=1}^7 i c_i(0)^{i-1} = -0.890246660831686, \quad (59)$$

$$f(0.2) = \sum_{i=0}^{11} a_i(0.2)^i = 0.238349795258861, \quad (60)$$

$$f(0.4) = \sum_{i=0}^{11} a_i(0.4)^i = 0.508035749401300, \quad (61)$$

$$f(0.6) = \sum_{i=0}^{11} a_i(0.6)^i = 0.754275169401941, \quad (62)$$

$$f(0.8) = \sum_{i=0}^{11} a_i(0.8)^i = 0.932055257058124, \quad (63)$$

$$g(0.2) = \sum_{i=0}^7 b_i(0.2)^i = 0.35104348534303, \quad (64)$$

$$g(0.4) = \sum_{i=0}^7 b_i(0.4)^i = 0.47111223484493, \quad (65)$$

$$g(0.6) = \sum_{i=0}^7 b_i(0.6)^i = 0.39553624042495, \quad (66)$$

$$g(0.8) = \sum_{i=0}^7 b_i(0.8)^i = 0.20550536616341, \quad (67)$$

$$\theta(0.2) = \sum_{i=0}^7 b_i(0.2)^i = 0.81495454573411, \quad (68)$$

$$\theta(0.4) = \sum_{i=0}^7 b_i(0.4)^i = 0.61756323216920, \quad (69)$$

$$\theta(0.6) = \sum_{i=0}^7 b_i(0.6)^i = 0.41199688810026, \quad (70)$$

$$\theta(0.8) = \sum_{i=0}^7 b_i(0.8)^i = 0.20410900521112, \quad (71)$$

$$\begin{aligned} f''(1) &= \sum_{i=2}^{11} i(i-1)a_i(1)^{i-2} \\ &= -3.70076904940068, \end{aligned} \quad (72)$$

$$\begin{aligned} f'''(1) &= \sum_{i=3}^{11} i(i-1)(i-2)a_i(1)^{i-3} \\ &= -4.30254152826803, \end{aligned} \quad (73)$$

$$g'(1) = \sum_{i=1}^7 i b_i(1)^{i-1} = -0.895870071804573, \quad (74)$$

$$\theta'(1) = \sum_{i=1}^7 i c_i(1)^{i-1} = -1.00257388455775 \quad (75)$$

By solving 28 algebraic equations (48)–(75) and 28 unknown coefficients of Equation (47), the semi-analytical solution of the ODEs (13)–(15) is derived as below:

$$\begin{aligned} f(\eta) &= -0.002502829121 \eta^{11} + 0.01924809861 \eta^{10} \\ &\quad - 0.05074212578 \eta^9 + 0.01612053153 \eta^8 \\ &\quad + 0.1921144887 \eta^7 - 0.3225151891 \eta^6 \\ &\quad - 0.06149744019 \eta^5 + 0.7229857992 \eta^4 \\ &\quad - 1.804978600 \eta^3 + 1.291767267 \eta^2 + \eta, \end{aligned} \quad (76)$$

$$\begin{aligned}
 g(\eta) = & 0.06857676653 \eta^7 + 0.01753848105 \eta^6 \\
 & - 1.167617948 \eta^5 + 3.291888403 \eta^4 \\
 & - 2.092260914 \eta^3 - 2.415694665 \eta^2 \\
 & + 2.297569877 \eta, \quad (77)
 \end{aligned}$$

$$\begin{aligned}
 \theta(\eta) = & 0.004878024475 \eta^7 - 0.03295822996 \eta^6 \\
 & + 0.03416874045 \eta^5 + 0.04751479451 \eta^4 \\
 & + 0.01708644167 \eta^3 - 0.1804431103 \eta^2 \\
 & - 0.8902466608 \eta + 1. \quad (78)
 \end{aligned}$$

By substituting the HAN solutions (76)-(78) into ODEs (13)-(15), it is possible to prove that 28 known coefficients make the solution more accurate than previous HAN solutions (44)-(46) with the 20 known coefficients.

Figures 5-7 demonstrates the substitution of the results of the analytical solutions (76)-(78) in the ODEs (13)-(15). The average values of $F(\eta)$ are 0.000244167582707404 (which is 197015.2466% more accurate than the previous solution), $G(\eta)$ is 0.000641019109384842 (which is 6948.985896% more accurate than previous solution), and $\Theta(\eta)$ is -0.000116302116405317 (which is 3787.698559% more accurate than previous solution). These average values are much less than previous ones, showing that increasing the number of terms in the analytical solutions improves their accuracy. The comparison of Figures 2-4 by Figures 5-7 proved that increasing the number of terms in the assumed solutions make the solution more accurate.

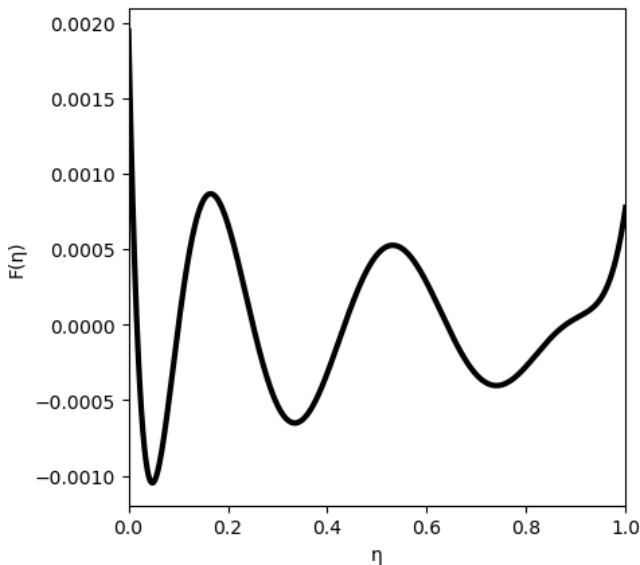


Fig. 5. The validity of $f(\eta)$ and $g(\eta)$ solutions.

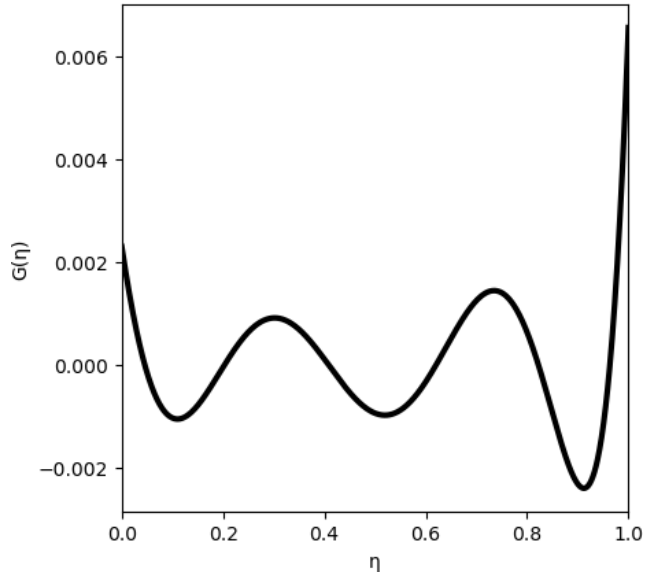


Fig. 6. The validity of $f(\eta)$ and $g(\eta)$ solutions.

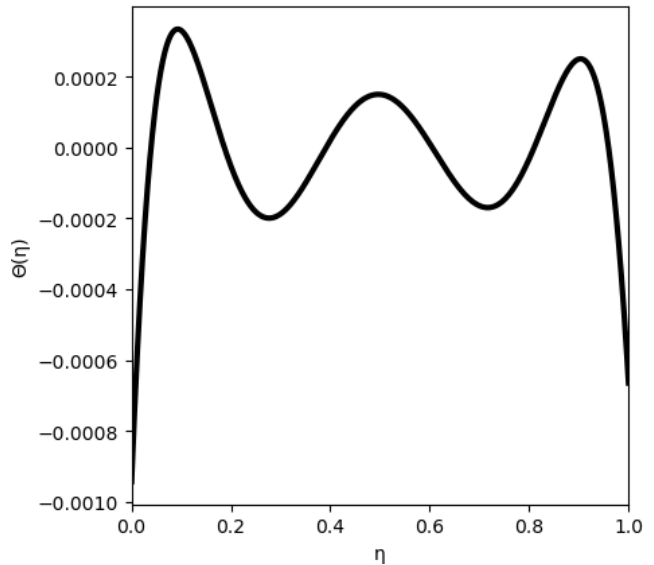


Fig. 7. The validity of $f(\eta)$ and $\theta(\eta)$ solutions.

4 Results and Discussion

4.1 Impact of changing rotation parameter

Changing the problem's physical structure (increasing or decreasing the distance between two walls) or dynamics of geometry (increasing/decreasing the angular velocity of the microchannel) will influence the dimensionless rotation parameter (λ). Therefore, the rotation parameter changes could be only from the fluctuations of the microchannel's height, the microchannel's angular velocity, or when they both change simultaneously.

It should be considered that the distance between the two walls has a direct relationship with the rotation parameter, and a small value for this parameter does not mean that the distance between the two walls is small but rather that the angular velocity of the channel is small. The results showed that when the rotation parameter changes from a very small value to a certain value, it only affects the fluid velocity quantities. The only quantity not affected by this parameter's change is the temperature quantity. Changing the rotation parameter affects all the velocity quantities in all three directions; however, each of these three velocities is affected by different amounts. According to Figures 8-10, as the rotation parameter changes from 0 to 40 while the other parameters remain fixed at $\phi_{\text{Al}_2\text{O}_3} = 0.25$, $\phi_{\text{Cu}} = 0.25$, $\rho_{\text{Al}_2\text{O}_3} = 3970$, $\rho_{\text{Cu}} = 8933$, $\rho_{\text{bf}} = 997.1$, $k_{\text{Al}_2\text{O}_3} = 40$, $k_{\text{Cu}} = 401$, $k_{\text{bf}} = 0.613$, $(c_p)_{\text{Al}_2\text{O}_3} = 765$, $(c_p)_{\text{Cu}} = 385$, $(c_p)_{\text{bf}} = 4179$, $\text{Re} = 2$, $\text{Pr} = 0.967411$, $N = 0.5$, $\theta_w = 1.5$, and $S = 1$, the average dimensionless hybrid nanofluid velocity along x -axis is decreases from 0.949973074 to 0.901305922, the average dimensionless hybrid nanofluid velocity along y -axis decreases from 0.582304917 to 0.505455989, while the average dimensionless hybrid nanofluid velocity along z -axis increases from 0 to 4.336965118.

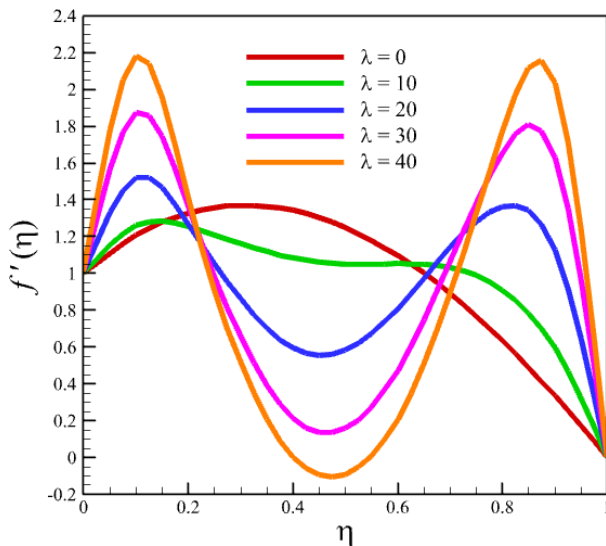


Fig. 8. The impact of rotation parameter on dimensionless hybrid nanofluid velocity along x -axis.

Therefore, it can be stated that increasing the rotation parameter has decreased the average velocity value in both x and y directions and increased the average velocity in the z direction. Figure 11 proves that changing the rotation parameter affects the fluid flow and cannot affect the temperature distribution. The skin friction coefficient changed from 2.076270305 to 26.62623289, which shows a significant influence of rotation param-

eters on fluid flow. However, according to Figure 11, the Nusselt number changes are negligible due to the changes of rotation parameter because it changes from 4.406015635 to 4.379470239.

4.2 Impact of changing suction / injection parameter

Considering the geometry of the upper wall –whether it is porous or solid, and whether the fluid enters the region between the two walls or flows above the upper wall – it has a direct effect on the value of the suction/injection parameter.

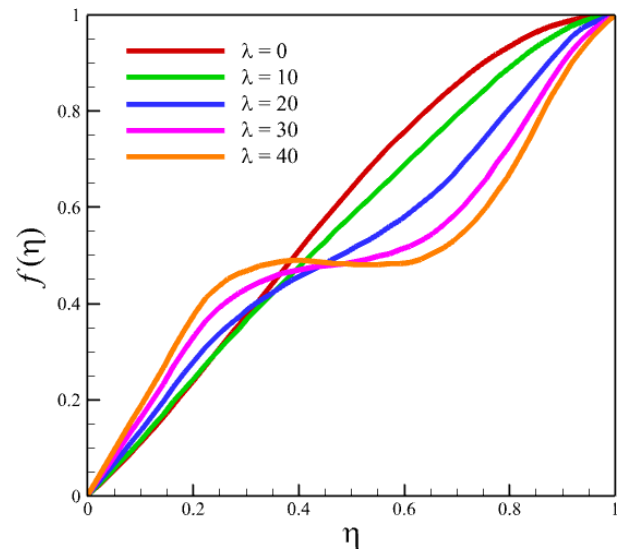


Fig. 9. The impact of rotation parameter on dimensionless hybrid nanofluid velocity along y -axis.

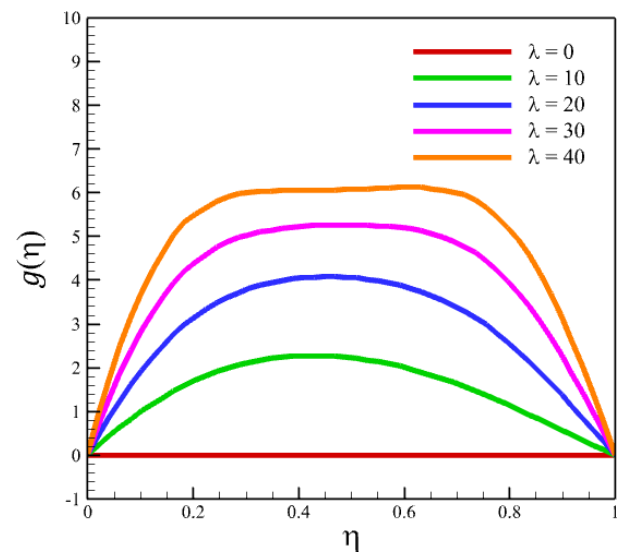


Fig. 10. The impact of rotation parameter on dimensionless hybrid nanofluid velocity along z -axis.

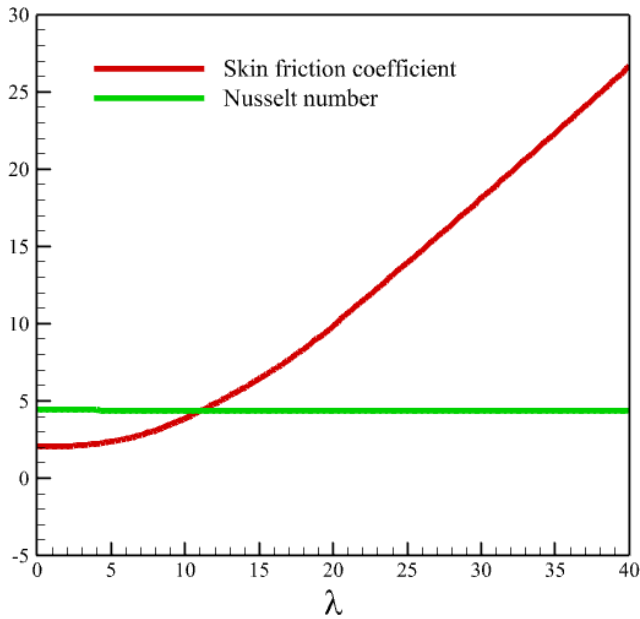


Fig. 11. The impact of rotation parameter on skin friction coefficient and Nusselt number for the bottom wall.

When ($S > 0$), suction of the hybrid nanofluid occurs at the upper wall. When the $S < 0$, injection from the top wall takes place. When the top wall is impermeable (solid), it corresponds to $S = 0$. The only factor that causes forced convection heat transfer in this problem is the presence of injection or suction from the boundary of the upper wall. When the upper wall is impenetrable (i.e., $S = 0$), then due to the temperature difference between the two surfaces, there is only natural heat transfer. According to the Figures 12-15, the changes in this parameter have a noticeable effect on the quantity of speed in three directions and temperature; however, the speed changes are more noticeable than the temperature. According to Figures 12-14, when the suction/injection parameter changes from 0 to 40 while the other parameters remain at $\phi_{\text{Al}_2\text{O}_3} = 0.25$, $\phi_{\text{Cu}} = 0.25$, $\rho_{\text{Al}_2\text{O}_3} = 3970$, $\rho_{\text{Cu}} = 8933$, $\rho_{\text{bf}} = 997.1$, $k_{\text{Al}_2\text{O}_3} = 40$, $k_{\text{Cu}} = 401$, $k_{\text{bf}} = 0.613$, $(c_p)_{\text{Al}_2\text{O}_3} = 765$, $(c_p)_{\text{Cu}} = 385$, $(c_p)_{\text{bf}} = 4179$, $\lambda = 2$, $\text{Pr} = 0.967411$, $\text{Re} = 2$, $N = 0.5$, and $\theta_w = 1.5$, the average dimensionless hybrid nanofluid velocity along x -axis changes from 0.05011907 to 35.55361994, the average dimensionless hybrid nanofluid velocity along y -axis changes from 0.072246588 to 23.83116986, the average dimensionless hybrid nanofluid velocity along z -axis changes from -0.041832591 to 0.593831969, and the average dimensionless hybrid nanofluid temperature changes from 0.526218038 to 0.20875792. Figures 12-15 proves that the change of suction/injection parameter affects the fluid flow and temperature, but Figure 16. shows

that the changes in fluid flow are much more significant than the temperature changes. According to Figures 16 and 17, when the suction/injection parameter changes from 0 to 40, the Nusselt number changes from 4.130100997 to 14.65008877. The Nusselt number changes are negligible compared to the skin friction coefficient because the skin friction coefficient has changed from -3.531045658 to 845.0782496.

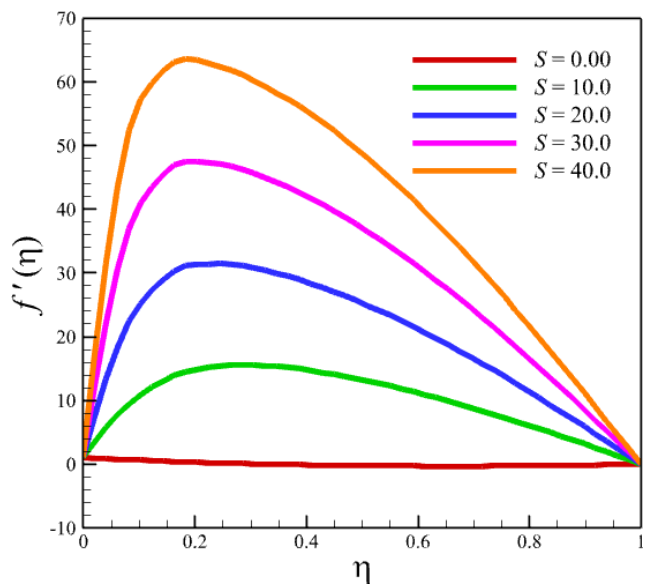


Fig. 12. The impact of suction/injection parameter on dimensionless hybrid nanofluid velocity along x -axis.

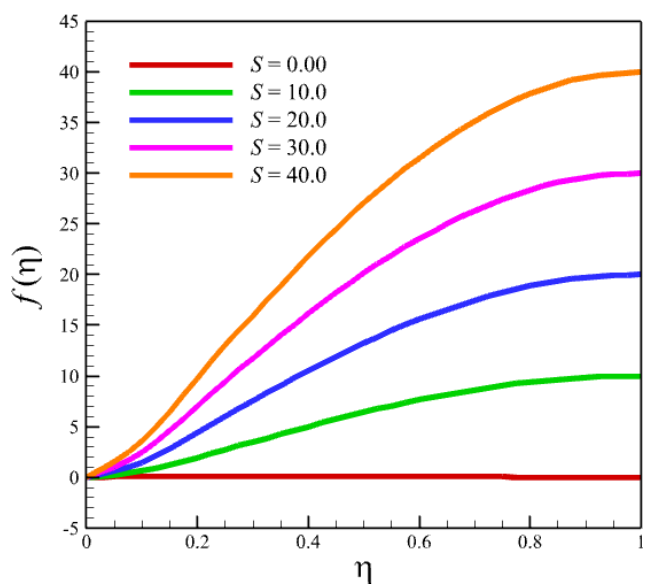


Fig. 13. The impact of suction/injection parameter on dimensionless hybrid nanofluid velocity along y -axis.

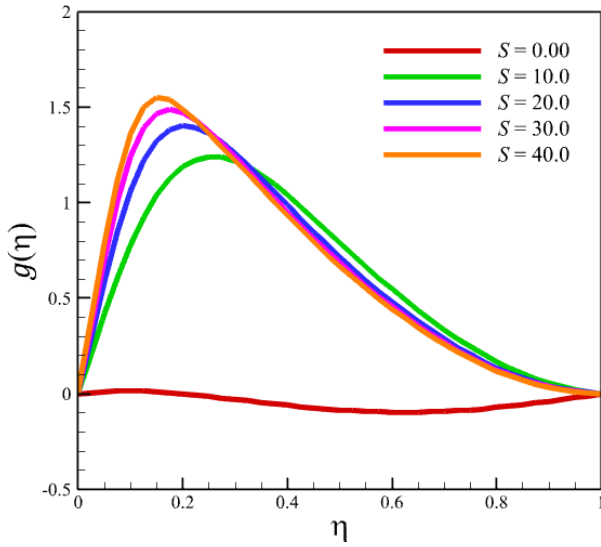


Fig. 14. The impact of suction/injection parameter on dimensionless hybrid nanofluid velocity along z -axis.

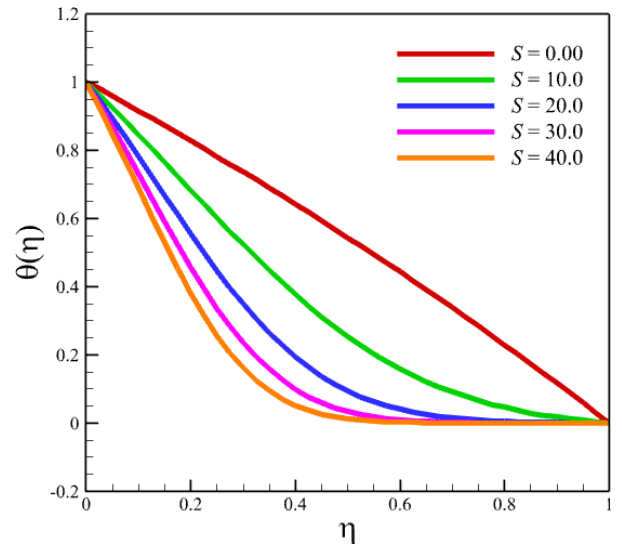


Fig. 15. The impact of suction/injection parameter on dimensionless hybrid nanofluid temperature distribution.

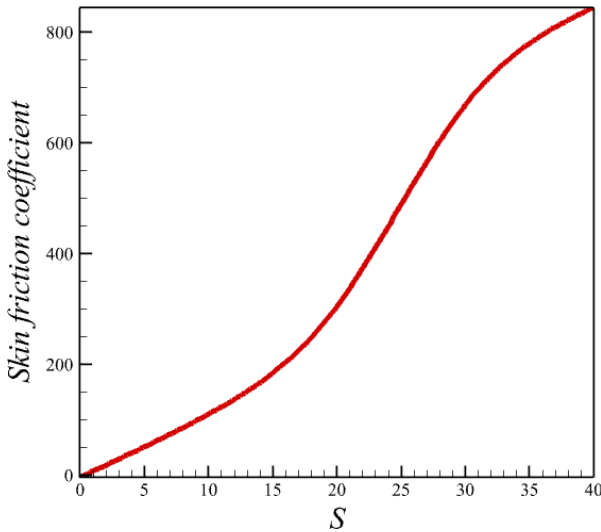


Fig. 16. The impact of suction/injection parameter on skin friction coefficient for the bottom wall.

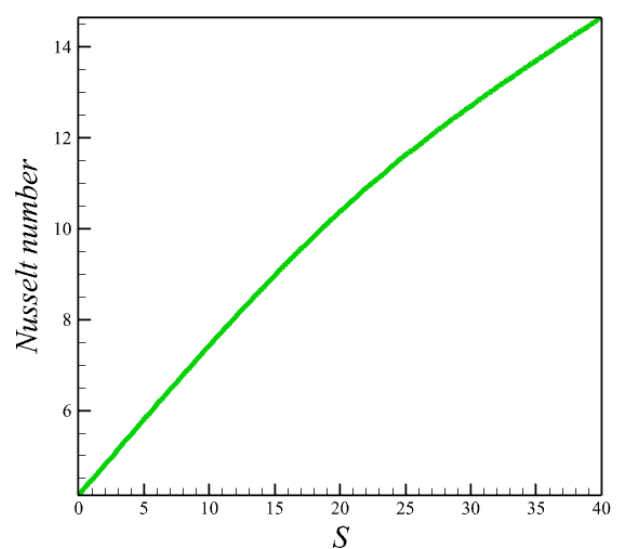


Fig. 17. The impact of suction/injection parameter on Nusselt number for the bottom wall.

4.3 Impact of changing Reynolds number

The changes of the Reynolds number from 10 to 50 will influence all quantities, such as fluid velocity along three axes, temperature distribution, skin friction coefficient, and the Nusselt number when $\phi_{\text{Al}_2\text{O}_3} = 0.25$, $\phi_{\text{Cu}} = 0.25$, $\rho_{\text{Al}_2\text{O}_3} = 3970$, $\rho_{\text{Cu}} = 8933$, $\rho_{\text{bf}} = 997.1$, $k_{\text{Al}_2\text{O}_3} = 40$, $k_{\text{Cu}} = 401$, $k_{\text{bf}} = 0.613$, $(c_p)_{\text{Al}_2\text{O}_3} = 765$, $(c_p)_{\text{Cu}} = 385$, $(c_p)_{\text{bf}} = 4179$, $\lambda = 2$, $\text{Pr} = 0.967411$, $N = 0.5$, $\theta_w = 1.5$, and $S = 1$. However, the in-

fluence of the Reynolds number is different in every quantity. Increasing the Reynolds number does not impact the fluid flow along x -axis and y -axis significantly. According to the Figures 18 and 19, the average dimensionless hybrid nanofluid velocity along x -axis changes from 0.94916661 to 0.946056479, the average dimensionless hybrid nanofluid velocity along y -axis changes from 0.596746732 to 0.611100794. However, simultaneously, fluid flow along z -axis and fluid temperature are more sensitive to the Reynolds number. According to the Figures 20 and 21, the average

dimensionless hybrid nanofluid velocity along z -axis changes from 0.09784131 to 0.022463717, the average dimensionless hybrid nanofluid temperature distribution changes from 0.430743758 to 0.241239845. Figure 22 shows the growth of the skin friction coefficient and Nusselt number due to the increase in the Reynolds number. According to Figure 22, when the Reynolds number increases, the skin friction coefficient changes from 3.452227132 to 7.185471981, and the Nusselt number changes from 6.241413391 to 12.66926207.

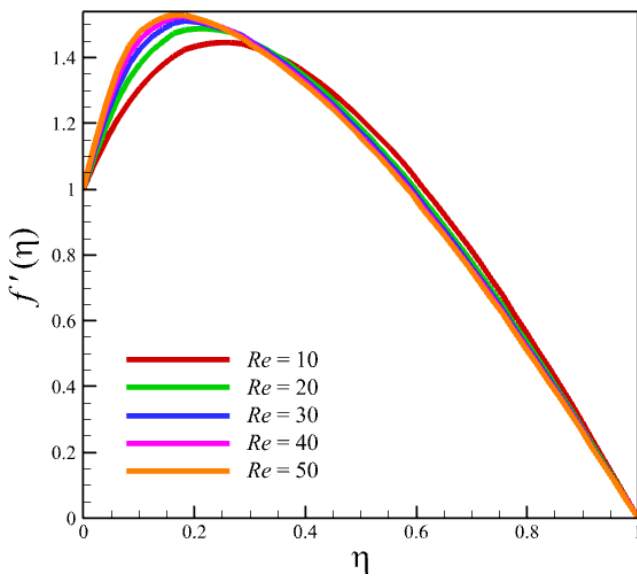


Fig. 18. The impact of Reynolds number on dimensionless hybrid nanofluid velocity along x -axis.

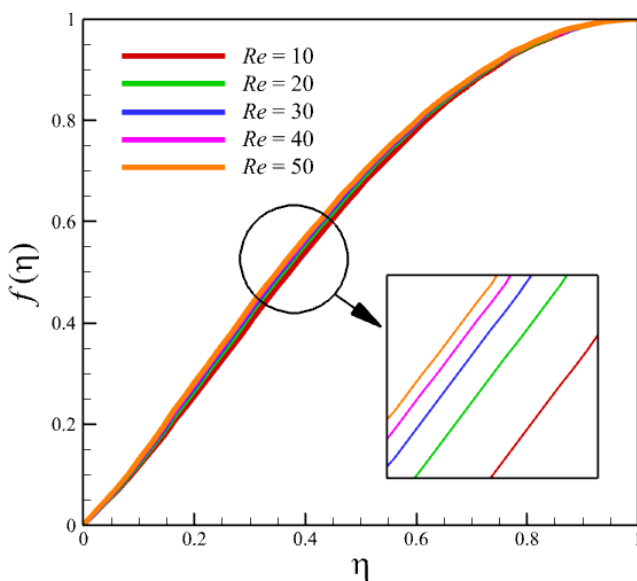


Fig. 19. The impact of Reynolds number on dimensionless hybrid nanofluid velocity along y -axis.

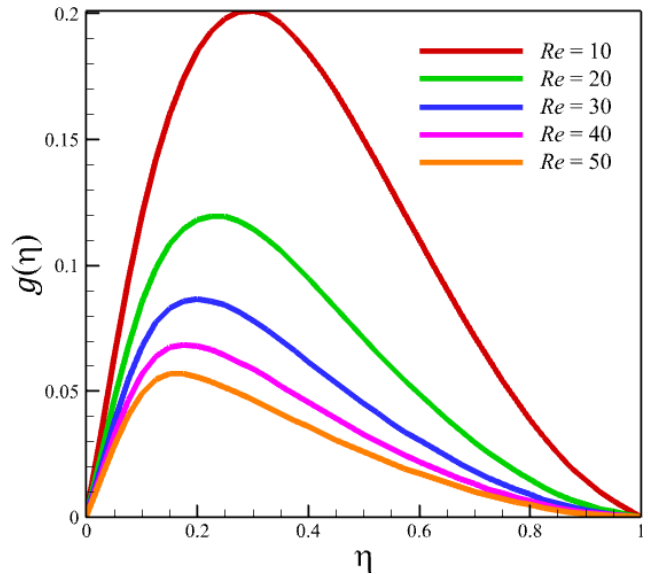


Fig. 20. The impact of Reynolds number on dimensionless hybrid nanofluid velocity along z -axis.

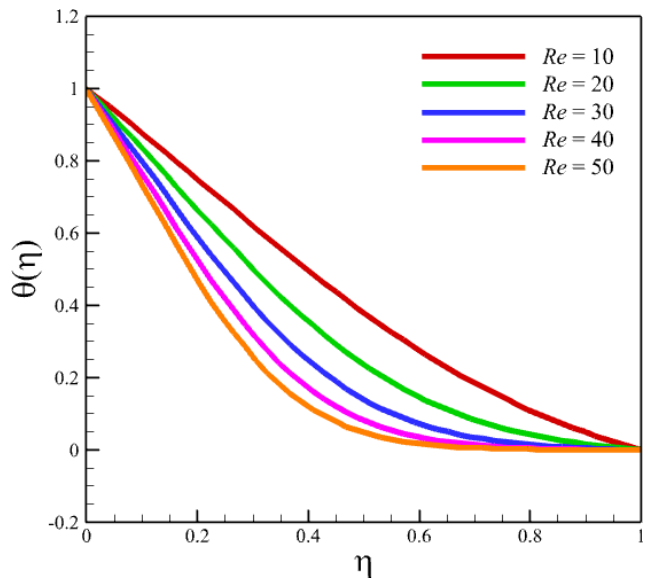


Fig. 21. The impact of Reynolds number on dimensionless hybrid nanofluid temperature distribution.

4.4 Impact of changing temperature ratio parameter

In this study, the temperature of the lower surface is always greater than or equal to the upper surface. By increasing the temperature of the lower wall or adjusting the upper wall temperature such that the ratio of the upper-wall temperature to the lower-wall temperature is greater than or equal to one, the temperature ratio parameter changes. The impact of the temperature ratio parameter is only on the fluid's temperature distribution and Nusselt number.

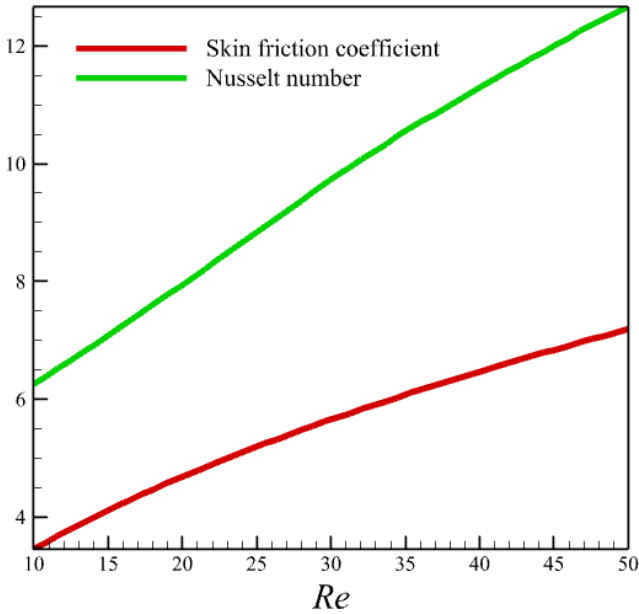


Fig. 22. The impact of Reynolds number on skin friction coefficient and Nusselt number for the bottom wall.

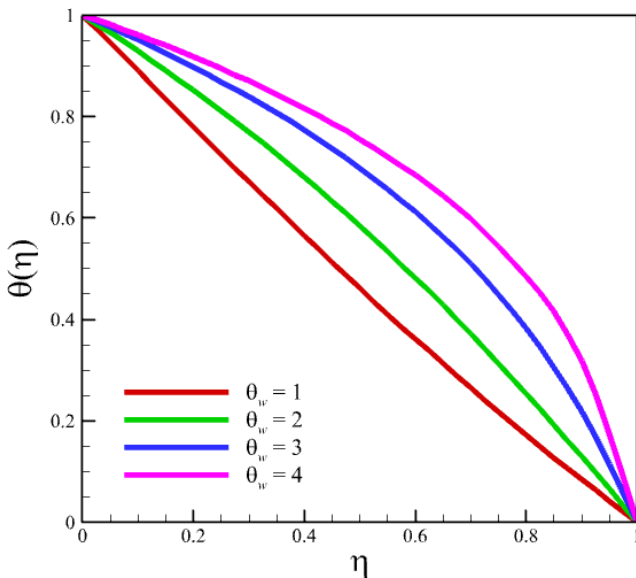


Fig. 23. The impact of temperature ratio parameter on dimensionless hybrid nanofluid temperature distribution.

According to the Figures 23 and 24, when the temperature ratio parameter changes from 1 to 4, the other parameters stays at $\phi_{\text{Al}_2\text{O}_3} = 0.25$, $\phi_{\text{Cu}} = 0.25$, $\rho_{\text{Al}_2\text{O}_3} = 3970$, $\rho_{\text{Cu}} = 8933$, $\rho_{\text{bf}} = 997.1$, $k_{\text{Al}_2\text{O}_3} = 40$, $k_{\text{Cu}} = 401$, $k_{\text{bf}} = 0.613$, $(c_p)_{\text{Al}_2\text{O}_3} = 765$, $(c_p)_{\text{Cu}} = 385$, $(c_p)_{\text{bf}} = 4179$, $\lambda = 2$, $\text{Pr} = 0.967411$, $\text{Re} = 2$, $N = 0.5$, and $S = 1$. Figure 23 shows temperature distribution changes from 0.477224804 to 0.673115582 when

the temperature ratio parameter increases from 1 to 4, and Figure 24 shows Nusselt number changes from 3.716535826 to 17.14280061. At the same time, the skin friction coefficients change from 2.137943694 to 2.137943592, proving that the temperature ratio parameter does not affect the fluid velocity.

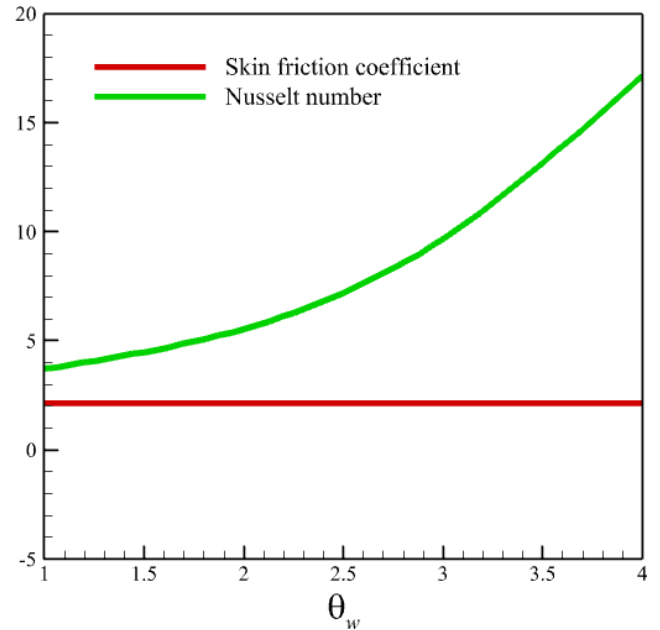


Fig. 24. The impact of temperature ratio parameter on skin friction coefficient and Nusselt number for the bottom wall.

4.5 Impact of changing solar radiation parameter

Changes in solar radiation parameters affected only the temperature distribution and the dimensionless Nusselt number at the lower wall, while having no influence on the fluid flow in different directions or on the skin friction coefficient. According to the Figures 25 and 26, when the solar radiation parameter changes from 1 to 100, the other parameters are $\phi_{\text{Al}_2\text{O}_3} = 0.25$, $\phi_{\text{Cu}} = 0.25$, $\rho_{\text{Al}_2\text{O}_3} = 3970$, $\rho_{\text{Cu}} = 8933$, $\rho_{\text{bf}} = 997.1$, $k_{\text{Al}_2\text{O}_3} = 40$, $k_{\text{Cu}} = 401$, $k_{\text{bf}} = 0.613$, $(c_p)_{\text{Al}_2\text{O}_3} = 765$, $(c_p)_{\text{Cu}} = 385$, $(c_p)_{\text{bf}} = 4179$, $\lambda = 2$, $\text{Pr} = 0.967411$, $\text{Re} = 2$, $\theta_w = 1.5$, and $S = 1$, only temperature and Nusselt number changes significantly. In Figure 25, the average value temperature changes from 0.528400804 to 0.585885889, and according to Figure 26, the average value of the Nusselt number changes from 5.756197375 to 273.8604886. The ineffectiveness of this parameter can be seen in Figure 26, where the skin friction coefficient does not change.

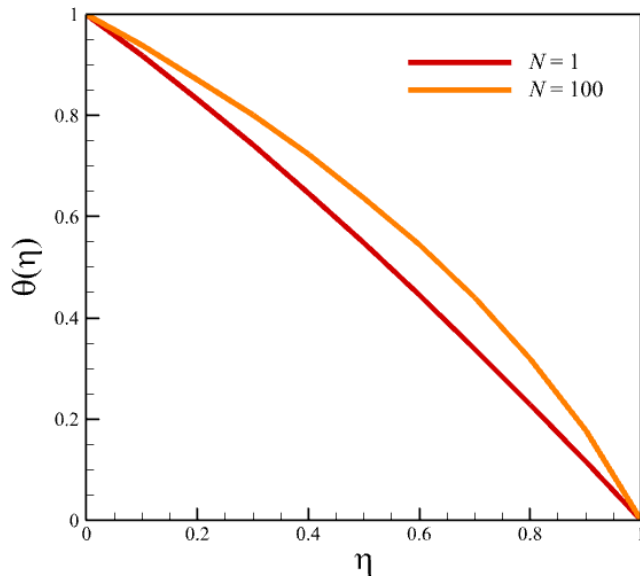


Fig. 25. The impact of solar radiation parameter on dimensionless hybrid nanofluid temperature distribution.

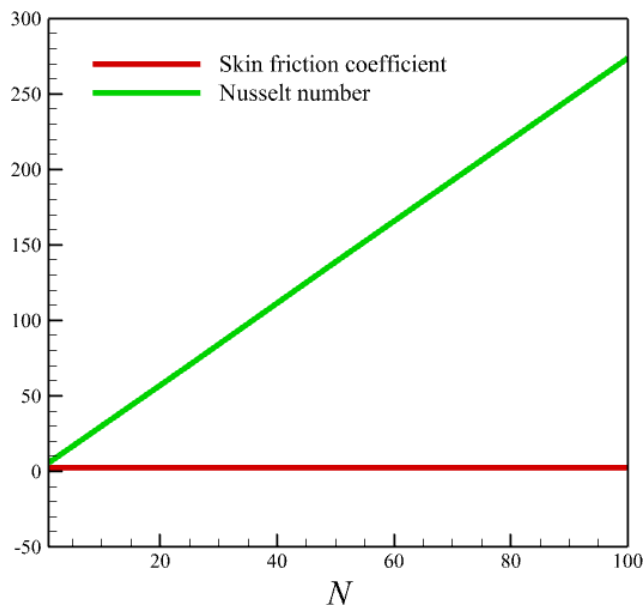


Fig. 26. The impact of solar radiation parameter on skin friction coefficient and Nusselt number for the bottom wall.

5 Conclusion

The fluid flow and heat transfer of a specific hybrid nanofluid named $\text{Al}_2\text{O}_3\text{-Cu}$ /water hybrid nanofluid inside a rotating microchannel in the presence of solar radiative energy were investigated. It is assumed that the temperature of the upper wall was lower than that of the lower wall. The differential equations governing

the physics of the problem were converted into ordinary differential equations using similarity transformations. These equations were then analytically solved using the HAN method. Finally, the following results are extracted from changing parameters depending on the physics of the problem, including the injection/suction parameter, rotation parameter, Reynolds number, temperature ratio parameter, and radiation parameter:

- Changes in the rotation parameter and its increase only have a significant effect on the fluid flow because it was observed that the surface friction coefficient changes significantly. Still, the Nusselt number does not change at all.
- Increasing the rotation parameter has caused a decrease in the average fluid velocity in both x and y directions and an increase in the average fluid velocity along the z -axis.
- By increasing the amount of fluid suction from the upper wall, the skin friction coefficient and Nusselt number values for the lower wall increase, but the increase of Nusselt number is much smaller than the skin friction coefficient.
- In the investigation of the suction/injection parameter, it was observed that by increasing the amount of fluid suction from the upper wall, the average fluid flow increases in all three directions, but the increase of this parameter causes a decrease in the average fluid temperature.
- Increasing the Reynolds number in the problem can only have a slight effect on the fluid velocity in the x and y directions. At the same time, it causes a significant decrease in the temperature and fluid velocity in the z -direction.
- The Nusselt number and skin friction coefficient for the lower wall increase significantly with increasing Reynolds number.
- With increasing the parameters of solar radiation and temperature ratio, Both the Nusselt number (for the lower wall) and average temperature of the fluid will increase.

Nomenclature

Quantity	Unit	symbol
A positive value	s^{-1}	b
Angular velocity of the microchannel	s^{-1}	Ω
Cartesian coordinate components	m	x, y, z
Cartesian velocity components	m s^{-1}	u, v, w
Density of Aluminum oxide nanoparticle	kg m^{-3}	$\rho_{\text{Al}_2\text{O}_3}$
Density of hybrid nanofluid	kg m^{-3}	ρ_{hnf}

Quantity	Unit	Symbol	Quantity	Unit	Symbol
Dimensionless hybrid nanofluid temperature	-	$\theta(\eta)$	Temperature of the upper boundary layer	K	T_0
Dimensionless hybrid nanofluid velocity along x -axis	-	$f'(\eta)$	Temperature ratio parameter	-	θ_w
Dimensionless hybrid nanofluid velocity along y -axis	-	$f(\eta)$	The angular velocity vector of the microchannel	s^{-1}	$\vec{\Omega}$
Dimensionless hybrid nanofluid velocity along z -axis	-	$g(\eta)$	The density of copper nanoparticles	$kg\ m^{-3}$	ρ_{Cu}
Dynamic viscosity of the base fluid	Pa s	ν_{bf}	The density of the base fluid	$kg\ m^{-3}$	ρ_{bf}
Dynamic viscosity of the hybrid nanofluid	Pa s	μ_{hnf}	The ratio of the density of hybrid nanofluid to the density of the base fluid	-	L_1
Height of the microchannel	m	h	The ratio of the dynamic viscosity of the hybrid nanofluid to the dynamic viscosity of the base fluid	-	L_4
Independent dimensionless similarity variable	-	η	The ratio of the thermal conductivity of the hybrid nanofluid to the thermal conductivity of the base fluid	-	L_3
kinematic viscosity of the base fluid	$m^2\ s$		The ratio of the volumetric heat capacity of the hybrid nanofluid to the volumetric heat capacity of the base fluid	-	L_2
Lower wall's heat transfer rate	$J\ s^{-1}$	q_w	The solid volume fraction of the aluminum oxide nanoparticle	-	$\phi_{Al_2O_3}$
Lower wall's Nusselt number	-	Nu	The solid volume fraction of the copper nanoparticle	-	ϕ_{Cu}
Lower wall's shear stress	$N\ m^{-2}$	τ_w	Thermal conductivity of the Aluminum oxide nanoparticle	$W\ m^{-1}\ K^{-1}$	$k_{Al_2O_3}$
Lower wall's skin friction coefficient	-	C_f	Thermal conductivity of the base fluid	$W\ m^{-1}\ K^{-1}$	k_{bf}
Mean absorption coefficient	m^{-1}	κ^*	Thermal conductivity of the copper nanoparticle	$W\ m^{-1}\ K^{-1}$	k_{Cu}
Modified pressure distribution of the hybrid nanofluid	Pa	P^*	Thermal conductivity of the hybrid nanofluid	$W\ m^{-1}\ K^{-1}$	k_{hnf}
Name of ODE (13)	-	$F(\eta)$	Volumetric heat capacity of the Aluminum oxide nanoparticle	$J\ K^{-1}\ kg^{-1}$	$(\rho c_p)_{Al_2O_3}$
Name of ODE (14)	-	$G(\eta)$	Volumetric heat capacity of the base fluid	$J\ K^{-1}\ kg^{-1}$	$(\rho c_p)_{bf}$
Name of ODE (15)	-	$\Theta(\eta)$			
Prandtl number	-	Pr			
Rate of the fluid injection/suction from the upper surface	$m\ s^{-1}$	v_w			
Reynolds number	-	Re			
Rotation parameter	-	λ			
Solar radiation parameter	-	N			
Solar radiative heat flux	$W\ m^{-2}$	q_r			
Specific heat capacity of the Aluminum oxide nanoparticle	$J\ m^3\ kg^{-2}\ K^{-1}$	$(c_p)_{Al_2O_3}$			
Specific heat capacity of the base fluid	$J\ m^3\ kg^{-2}\ K^{-1}$	$(c_p)_{bf}$			
Specific heat capacity of the copper nanoparticle	$J\ m^3\ kg^{-2}\ K^{-1}$	$(c_p)_{Cu}$			
Specific heat capacity of the hybrid nanofluid	$J\ m^3\ kg^{-2}\ K^{-1}$	$(c_p)_{hnf}$			
Stefan-Boltzmann constant	$W\ m^{-2}\ K^{-4}$	σ^*			
Stretching rate of the lower wall	$m\ s^{-1}$	U_w			
Suction/injection parameter	-	S			
Temperature	K	T			
Temperature of the lower boundary layer	K	T_h			

Quantity	Unit	Symbol
Volumetric heat capacity of the copper nanoparticle	$\text{J K}^{-1} \text{kg}^{-1}$	$(\rho c_p)_{\text{Cu}}$
Volumetric heat capacity of the hybrid nanofluid	$\text{J K}^{-1} \text{kg}^{-1}$	$(\rho c_p)_{\text{hnf}}$

Declaration of Interest

No conflict of interest has been declared by the authors.

Author's Contributions

All authors contributed to this study.

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Data Availability

This study is theoretical in nature, and therefore, no datasets were generated or analyzed.

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